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THE MAXIMUM PRINCIPLE OF L.S. PONTRYAGIN IN OPTIMAL-SYSTEM THEORY. PART III*

L.I. Rozonoér

(Moscow)

The paper deals with questions related to the proof and employment of L.S. Pontryagin's maximum principle in optimal-system theory. Certain new results are also contained in this work. The first part of the work deals with the optimization problem for the case of free trajectory right ends. In the second part, the maximum principle is formulated for a more general type of boundaryconditions. In the third part, the connection between the method of dynamic programing and the maximum principle is established, a method of solving the optimization problem in discrete linear systems is given, and a number of considerations are presented concerning the use of the maximum principle for solving a definite class of problems which are related to the theory of dynamic accuracy of control systems.

4. Relationship of L.S. Pontryagin's Maximum Principle to R. Bellman's Dynamic Programing Method

We show, in this section, that with certain requirements of a general character there exists a close relationship between the equations deriving from the maximum principle and the corresponding equations from the theory of dynamic programing [1-3].

a) The Optimality Principle

Initially, we give a brief presentation of the idea of the dynamic programing method, using as an example a problem analogous to that considered by us in section 2 of part I. Let it be required to control the system described by the equations

$$\dot{x}_i = f_i(x, u, t), \qquad i = 1, \ldots, n, \qquad u(t) \in U, \tag{4.1}$$

such that a given function, $F[x(T)]$ of the state of the system at a fixed time T will be minimized, given that the system initially starts at point x^0 at time t_0. We denote the optimal control by $u^*(t) \equiv u^*(x^0, t_0, t)$ and the corresponding optimal trajectory by $x^*(t) \equiv x^*(x^0, t_0, t)$. We now consider the position on the optimal trajectory of the representative point $x' = x^*(x^0, t_0, t')$ at some moment of time $t = t'$, $t_0 < t' < T$, and we pose for ourselves the same problem as was formulated above, but with the initial time and initial state now taken to be, respectively, t' and x'. It is now necessary to find the optimal control $u^{**}(t) \equiv u^{**}(x', t', t)$ which minimizes the functional $F[x(T)]$ if the system is in state x' at initial time t'.

It is easily seen that the control $u^*(t)$, which is optimal on the time interval (t_0, T) in the problem with initial state x^0, t_0, must also be optimal on the interval (t', T) in the problem with initial state x', t', i.e., the values of the functional $F[x(T)]$ for the controls $u^*(t)$ and $u^{**}(t)$ must be equal.

*Parts I and II of this work were published, respectively, in Automation and Remote Control 10 and 11 (1959). [See English translation].

Indeed, if this assertion were not correct, and the functional's value for control $u^{**}(t)$ were less than its value for control $u^*(t)$ with the system initially at x', t', then control $u^*(t)$ could always be "improved" in the problem with initial state x^0, t_0, by replacing it by the control

$$v(t) = \begin{cases} u^*(t) & t_0 \leqslant t < t', \\ u^{**}(t) & t' \leqslant t \leqslant T, \end{cases}$$

which would first take the system to point x' and thereafter to the corresponding final state. But since control $u^*(t)$ is, by definition, optimal in the problem with initial state x^0, t_0 and cannot be improved, we arrive at a contradiction, and thus prove our assertion. This very simple fact is the essence of the so-called "optimality principle" which, following R. Bellman ([1], p. 83), we formulate in the following way.

An optimal control has the property that, whatever the initial state and initial control are, the remaining control must constitute an optimal control with regard to the state resulting from the first stage of the control.

Despite its simplicity, the optimality principle in many cases allows one to obtain an equation which ultimately defines the optimal control.

b) Basic Equation of the Dynamic Programing Method

We now derive the proper equation for the problem we are considering. We denote by $S_T(x^0, t_0)$ the value of the functional provided by optimal control $u^*(t)$ ($t_0 \leq t \leq T$) with the system initially at x^0, t_0. In our case, $S_T(x^0, t_0) \equiv F[x^*(x^0, t_0, T)]$. By varying the initial data, x^0, t_0, of the problem, we obtain a function $S_T(x^0, t_0)$ on the points of the $(n + 1)$-dimensional space $L(x_1^0, \ldots, x_n^0, t)$. When it leads to no ambiguities, we shall omit the superscript 0 and write the function S in the form $S_T(x, t)$.

As before, we consider the state x', t' on the optimal trajectory $x^*(t)$ where $x^*(t_0) = x^0$. When motion is along an optimal trajectory from state x^0, t_0, the functional assumes the value $S_T(x^0, t_0)$. By virtue of the optimality principle, controls $u^*(t)$ and $u^{**}(t)$ on the segment (t', T) correspond to one and the same optimal value of the functional, equal to $S_T(x', t')$. But since the functional is defined only by the terminal position of the representative point (at time $t = T$), we have that

$$S_T(x^0, t_0) = S_T(x', t'). \tag{4.2}$$

We consider the optimal control $u^*(t)$ on the segment $[t_0, t']$. It is obvious that, for optimality of the control $u^*(t)$, it is necessary that, to the point x' at time t', there correspond the least value of the function $S_T(x', t')$ in comparison to all other values which could be obtained for all other states reachable from x^0, t_0, at time t' by means of admissible controls $u(t) \in U$. Taking (4.2) into account, we can write this requirement in the following way:

$$S_T(x^0, t_0) = \min_{\substack{u(t) \in U \\ t_0 < t < t'}} S_T(x', t'), \tag{4.3}$$

bearing in mind for this that x' is a functional on $u(t)$ and, moreover, depends on x^0, t_0.

We now set $t' = t_0 + \tau$, where τ will be considered to be quite small. Then, bearing in mind that $x^*(t_0) = x^0$ and $\dot{x}_i^*(t_0) = f_i[x^0, t_0, u^*(t_0)]$, we write

$$x_i' \equiv x_i^*(t_0 + \tau) = x_i^0 + \tau f_i[x^0, t_0, u^*(t_0)] + \varepsilon,$$

where ϵ is of a higher order of smallness than τ. By assuming the existence and continuity of the partial derivatives of the function $S_T(x, t)$, we can write

$$S_T(x', t') = S_T(x^0, t_0) + \sum_1^n \frac{\partial S_T(x^0, t_0)}{\partial x_i^0}(x_i' - x_i^0) + \frac{\partial S_T(x^0, t_0)}{\partial t_0}(t' - t_0) + \beta,$$

where β is again small in comparison with τ. By using the previous formula for x_i', and by substituting the expression for $S_T(x',t')$ in (4.3), we get

$$S_T(x^0, t_0) = \min_{\substack{u(t)\in U\\ t_0<t<t_0+\tau}} \left[S_T(x^0, t_0) + \tau \sum_1^n \frac{\partial S_T(x^0, t_0)}{\partial x_i^0} f_i[x^0, t_0, u^*(t_0)] + \right.$$

$$\left. + \tau \frac{\partial S_T(x^0, t_0)}{\partial t_0} + \gamma \right].$$

The only terms depending on control u(t) are those containing the f_i and, in addition, the quantity γ which is of a higher order of smallness than τ. Therefore, the terms $S_T(x^0,t_0)$ and $\tau \frac{\partial S_T(x^0, t_0)}{\partial t_0}$ can be "taken out from in front of" the "min" sign. By simplifying and dividing by τ, we get

$$\frac{\partial S_T(x^0, t_0)}{\partial t_0} = - \min_{\substack{u(t)\in U\\ t_0<t<t_0+\tau}} \left[\sum_1^n \frac{\partial S_T(x^0, t_0)}{\partial x_i^0} f_i[x^0, t_0, u^*(t)] + \frac{\gamma}{\tau} \right].$$

This equation, just as (4.3), defines the choice of $u^*(t)$ on the segment $[t_0,t']$. By now letting τ approach zero, which also reduces the $[t_0, t_0 + \tau]$ segment to zero, we obtain an equation which determines the choice of $u^*(t)$ at the single point $t = t_0$. Therefore, by letting $u^*(t_0) = u^0$, and bearing in mind that $\lim_{\tau\to 0} \frac{\gamma}{\tau} = 0$, we obtain, finally,

$$\frac{\partial S_T(x^0, t_0)}{\partial t_0} = \min_{u'\in U} \sum_1^n \frac{\partial S_T(x^0, t_0)}{\partial x_i^0} f_i(x^0, t_0, u^0). \tag{4.4}$$

This relationship is valid for any x^0,t_0. Therefore, the subscript (and superscript) 0 will henceforth be frequently omitted. Furthermore, by using the evident relationship $\max(-\psi) = -\min \psi$ (where ψ is an arbitrary function), we will use Eq. (4.4) in the form

$$\frac{\partial S_T(x, t)}{\partial T} = \max_{u\in U} \sum_1^n \left(- \frac{\partial S_T(x, t)}{\partial x_i} \right) f_i(x, t, u). \tag{4.5}$$

Equation (4.5) is essentially a specific partial differential equation which can be integrated if the corresponding boundary conditions are given. The quantity u can be eliminated from (4.5) if one makes use of the requirement according to which the sum $\sum_1^n \left(- \frac{\partial S_T(x, t)}{\partial x_i} \right) f_i(x, t, u)$ must be maximal for u for any values of x,t and $\partial S_T/\partial x_i$. The elimination of u from (4.5) is carried out in exactly the same way as the elimination of u(t) from the function H in L.S. Pontryagin's maximum principle (cf. section 2, part I and section 3, part II). For this, u is expressed in terms of x,t and $\partial S_T/\partial x_i$:

$$u = \gamma\left(x, t, \frac{\partial S_T}{\partial x_i} \right). \tag{4.6}$$

For example, in the case of the equation

$$\frac{\partial S_T}{\partial t} = \max_u \left[- \frac{\partial S_T}{\partial x_1}(ux_1 + x_2) - \frac{\partial S_T}{\partial x_2} u^2 \right]$$

with the assumption that $\partial S_T/\partial x_2 > 0$, the maximum of the expression $- \frac{\partial S_T}{\partial x_1} x_1 u - \frac{\partial S_T}{\partial x_2} u^2 - \frac{\partial S_T}{\partial x_1} x_2$ as a function

of $\underline{u}$ is attained when $u = -\frac{1}{2} x_1 \dfrac{\frac{\partial S_T}{\partial x_1}}{\frac{\partial S_T}{\partial x_2}}$, so that after $\underline{u}$ is eliminated we obtain the equation

$$\frac{\partial S_T}{\partial t} = x_1^2 \frac{\left(\frac{\partial S_T}{\partial x_1}\right)^2}{4 \frac{\partial S_T}{\partial x_2}} - \frac{\partial S_T}{\partial x_1} x_2.$$

The boundary conditions for Eq. (4.5) are very simply found: for t = T, state x,t is simultaneously initial and final, and the functional's value does not depend on the control, and equals

$$S_T(x, T) \equiv F(x_1, \ldots, x_n). \tag{4.7}$$

Now, the solution of Eq. (4.5) can be determined. Indeed, from (4.6) the derivatives $\partial S_T(x,T)/\partial x_i$ are determined for each value of $\underline{x}$ and, consequently, the right member of (4.5) is also determined. In the same way, we learn the value of the partial derivative $\partial S_T(x,T)/\partial t$ and, for sufficiently small τ, we can find the value of $S_T(x, T-\tau)$ in the form

$$S_T(x, T-\tau) \approx S_T(x, T) - \frac{\partial S_T(x, T)}{\partial t}\tau.$$

Thus, moving step-by-step, we can determine the function $S_T(x,t)$ for any value of $\underline{t}$. If the function $S_T(x,t)$ is known then, by means of Eq. (4.6), $\underline{u}$ is also determined as a function of x,t:

$$u = \varphi(x, t). \tag{4.8}$$

Thus, we have uncovered the partial differential equation and the corresponding boundary conditions*

$$\frac{\partial S_T(x, t)}{\partial t} = \max_{u \in U} \sum_1^n \left(-\frac{\partial S_T(x, t)}{\partial x_i}\right) f_i(x, u, t), \tag{4.9}$$

$$S_T(x, T) \equiv F(x),$$

which allow us to determine the function $S_T(x,t)$ and the min-optimal control as functions of x,t. An analogous equation can be obtained for the case when one is seeking a maximum of the functional $F[x(T)]$:

$$\frac{\partial S_T(x, t)}{\partial t} = \min_{u \in U} \sum_1^n \left(-\frac{\partial S_T(x, t)}{\partial x_i}\right) f_i(x, u, t), \tag{4.10}$$
$$S_T(x, T) \equiv F(x).$$

The arguments leading to Eqs. (4.9) and (4.10) can in no way be considered as proofs. They should be considered as strictly heuristic, allowing us to surmise how the problem must be solved. However, as will be shown below, with certain assumptions the validity of the final results [i.e., an equation of the type of (4.9) or (4.10)] can be proved.

c) Relationship of the Maximum Principle to the Dynamic Programing Method

The structure of the right members of Eqs. (4.9) and (4.10) recalls the structure of the function

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* In the analogous equations [1], the partial derivatives were takenwith respect to T, rather than with respect to the initial time $\underline{t}$, which leads to a change in sign in the formulas.

$H = \sum_1^n p_i f_i\ (x,\ u,\ t)$ in the maximum principle. We convince ourselves that this similarity is not a casual one by theorem 6, given below, which is formulated for the case of the problem which we considered in section 2, part I [i.e., the problem of optimizing the functional $S = \sum_1^n c_i x_i\ (T)$].

We introduce the following notation: L is an n-dimensional space with coordinate axes $(x_1, \ldots, x_n, t)$; (x,t) is a point of space L; $u^*(t) \equiv u^*(x^0, t_0, t)$ is a min-optimal (max-optimal) control when the system's representative point is at position x^0 at initial time $t = t_0$; $x^*(t) \equiv x^*(x^0, t_0, t)$ is the corresponding optimal trajectory [so that $x^*(t_0) = x^0$]; $S_T(x,t)$ is the value of the functional $S = \sum_1^n c_i x_i\ (T)$ on the min-optimal (max-optimal) control if, at time $\underline{t}$, the system is found at point $\underline{x}$.

Theorem 6. In the problem of the min-optimization (max-optimization) of the functional $S = \sum_1^n c_i x_i\ (T)$ let the function $S_T(x,t)$ be continuous and continuously differentiable in the region Γ of space L.

Then:

1) For all $\underline{t}$ for which $(x^*(t),t) \in \Gamma$, the min-optimal (max-optimal) control $u^*(t)$ satisfies the maximum (minimum) condition with respect to $p(t) = [p_1(t), \ldots, p_n(t)]$, where

$$p_i\ (t) \equiv -\frac{\partial S_T\ [x^*\ (t),\ t]}{\partial x_i}, \qquad i = 1, \ldots, n,$$

and where

$$\frac{\partial S_T\ [x^*\ (t),\ t]}{\partial t} = H\ [x^*\ (t),\ p\ (t),\ u^*\ (t),\ t] = \sum_1^n p_i\ (t)\ f_i\ [x^*\ (t),\ u^*\ (t),\ t];$$

2) The function $S_T(x,t)$ satisfies, in Γ, the partial differential equation

$$\frac{\partial S_T\ (x,\ t)}{\partial t} = \max_{u \in U} \sum_1^n \left(-\frac{\partial S_T\ (x,\ t)}{\partial x_i}\right) f_i\ (x,\ u,\ t).$$

$$\left\langle \frac{\partial S_T\ (x,\ t)}{\partial t} = \min_{u \in U} \sum_1^n \left(-\frac{\partial S_T\ (x,\ t)}{\partial x_i}\right) f_i\ (x,\ u,\ t) \right\rangle,$$

where $S_T\ (x,\ T) \equiv \sum_1^n c_i x_i$, if $(x,T) \in \Gamma$.

Theorem 6 has a graphic geometric interpretation. In section 2, part I, we recalled that the vector $p(t)$, with respect to which optimal control $u(t)$ satisfies the maximum (minimum) condition, defines the direction of "maximum acceleration" of the system in phase space X. As follows from theorem 6, this direction, at each fixed moment of time $\underline{t}$, is determined in its turn by the gradient of the function $S_T(x,t)$ in phase space X. It thus turns out that, at each fixed moment of time, whatever the position of the representative point, the optimal control tends to "accelerate" the latter in the direction defined by the gradient of the function $S_T(x,t)$.

An analogous partial differential equation can also be obtained in the more general problem considered in section 3, part II. However, the boundary conditions here are significantly more complicated, and we shall not

consider them here. In any event, the relationship of the function p(t) to the gradient of the function S is retained even in this more complicated case.*

d) Analogy with the Equations of Analytical Mechanics

The analogy of the equations deriving from L.S. Pontryagin's maximum principle with the canonic equations of analytical mechanics was already cited in section 2, part I and section 3, part II. The establishment of the relationship between the maximum principle and the dynamic programing method shows that this analogy is very profound. The equations deriving from the dynamic programing method are completely analogous with the Hamilton-Jacobi partial differential equations, whereby the function S(x,t) plays the role of the action function. It thus turns out that the connection between the equations of the maximum principle and the equations of the dynamic programing method is analogous to the relationship between the Hamilton canonical equations and the Hamilton-Jacobi equations. The sole difference is that, in optimal system theory, the "control" is explicitly introduced, whereas it is eliminated in analytical mechanics, and is expressed in terms of the coordinates, their derivatives and time. As physical analogies of the control we can use, in particular, the velocities of the system's mass points.

e) The Role of the Requirement of Continuity

In the present work we have always considered problems in which the set U, defining the limitations on the class of admissible controls, does not depend on the running value of the coordinates. Otherwise the maximum principle, in the form in which we have formulated it, turns out to be inapplicable, whereas the equations of the dynamic programing method remain valid in general. This circumstance is explained as follows. The vector p(t) which enters into the formulation of the maximum principle has always been considered by us as a continuous function of time. But if the derivatives $\partial S_T(x,t)/\partial x_i$ are not continuous in the entire space L and, moreover, if the optimal trajectory $(x(t),t)$, $t_0 \leq t \leq T$ intersects a surface of discontinuity of the functions $\partial S_T(x,t)/\partial x_i$ in L, then the functions $p_i(t) \equiv -\partial S_T(x(t),t)/\partial x_i$ also experience a discontinuity. Therefore, it is necessary to generalize the maximum principle so as to allow us to consider discontinuous (in particular, piecewise-continuous) pulse functions. Such a generalization was made by R.V. Gamkrelidze [4] for the case of optimization with account taken of limitations placed on the system coordinates. The conditions for jumps in the functions $p_i(t)$ were formulated in [4].

Thus, the requirement connected with the continuity of the function $S_T(x,t)$ and its derivatives has a nonformal character. If we follow the mechanical analogy, the requirement that p(t) be continuous corresponds to the requirement of no impulse skips, i.e., the absence of "impact" interaction.

Whether or not the function $S_T(x,t)$ itself is continuous is particularly important. The fact of the matter is that the corresponding partial differential equation is valid, generally speaking, only in the regions of continuity of the functions $\partial S_T/\partial x_i$ in the space L. The condition that $S_T(x,t)$ be continuous allows one to "join" the solutions at the surfaces of discontinuity of the derivatives $\partial S_T/\partial x_i$. The author knows of only one work [5] investigating such questions in which continuity is proven for the optimal values of the functional (time control) in the linear problem of temporal optimization with limitations on the control of the type $|u_k| \leq 1$.

We note further that the existence of singular controls (cf. section 3, part II) is also related to the presence of discontinuities of the derivatives $\partial S_T/\partial x_i$. A singular trajectory occurs precisely along a surface of discontinuity.

f) The Synthesis Problem

As has been mentioned, the knowledge of the function $S_T(x,t)$ allows one to find the control $\underline{u}$ in the form of function (4.8), which rapidly gives a complete solution to the synthesis problem. However, solution of partial differential Eqs. (4.9) and (4.10) in closed form is possible only in the most elementary cases. In practice, therefore, one should attempt to solve the synthesis problem by means of the dynamic programing method only if one has access to a computer. The development of convenient machine algorithms for solving these equations is, therefore, of importance.

*In the general case, the surface S(x,t) = const is the geometric locus of the points which give the same value to the functional when the optimal trajectory begins with them. In particular, in the case of maximizing speed of response, the corresponding surfaces T(x) = const are the "isochrone" surfaces first considered by A.Ya. Lerner.

5. Concerning Optimal Processes in Discrete Systems

In this section we shall consider several optimization problems in discrete automatic control systems, described by finite difference equations. For this, we assume time to be "discrete," i.e., time takes the values $0, \tau, \ldots, m\tau, \ldots$ where τ is some constant having the dimensionality of time. The control processes in such systems can be described by the following system of finite difference equations:

$$x_i^{m+1} = x_i^m + \tau f_i(x_1^m, \ldots, x_n^m, u_1^m, \ldots, u_r^m, m\tau) \quad (i = 1, \ldots, n). \tag{5.1}$$

The symbols x_i^m, u_k^m are the abbreviated notations for the variables x_i and u_k considered at time $m\tau$, i.e., $x_i^m = x_i(m\tau)$, $u_k^m = u_k(m\tau)$. In the sequel we shall consider $\underline{m}$ as the time variable, assuming the integral values 0, 1, 2, . . .

In system (5.1) we have, as before, denoted by x_i $(i = 1, \ldots, n)$ and u_k $(k = 1, \ldots, r)$ the generalized system coordinates and, respectively, the controlling actions. By introducing the vectors $x^m = (x_1^m, \ldots, x_n^m)$ and $u^m = (u_1^m, \ldots, u_r^m)$, we can write system (5.1) briefly as

$$x^{m+1} - x^m = \tau f(x^m, u^m, m).$$

We shall consider as given the value of vector x^m at time $m = 0$, and shall denote it by x^0. The space $X(x_1, \ldots, x_n)$ in which the generalized system coordinates can vary is called the phase space. A "trajectory" of a discrete system in the phase space is a sequence of isolated points.

As before, we shall call the vector u^m, considered as a vector function of time $\underline{m}$, the control. The limitations on the controlling actions are expressed by requirements in accordance with which, at each moment of time $\underline{m}$, the control u^m must lie in a closed set U of space $R(u_1, \ldots, u_r)$:

$$u^m \in U \quad (m = 0, 1, 2, \ldots). \tag{5.2}$$

With respect to system (5.1), we can pose various optimization problems, completely analogous to those formulated for continuous systems in section 1, part I. The sole difference is that, in the proper places, the integrals over the continuous time $\underline{t}$ are replaced by sums over the discrete time $\underline{m}$, and derivatives with respect to $\underline{t}$ are replaced by the differences of variable values at neighboring instants of time $\underline{m}$. A wide class of optimization problems (in complete analogy with the problems for continuous systems) can be reduced to the following basic problem. From the set of admissible controls $u^m \in U$ which take the system from point x^0 to a fixed closed set G of phase space X, it is required to so choose the control u^m $(m = 0, 1, \ldots, M-1)$ that the sum $S = \sum_1^n c_i x_i^M$ at a given moment of time $m = M$, assumes a minimum (maximum) value.

The control time here is assumed fixed. Problems in which the time is not given beforehand will not be considered further here.

We remark that the problem formulated above is essentially the problem of minimizing (maximizing) the function S with respect to rM independent variables $u_r^0, \ldots, u_2^0; u_1^1, \ldots, u_r^1; u_1^{M-1}, \ldots, u_r^{M-1}$. Indeed, system (5.1) is a system of recurrence relationships which allow one to determine successively the values of the vectors $x^1, x^2, \ldots, x^M$ in the form of functions of the coordinates of the vectors $u^0, u^1, \ldots, u^{M-1}$ if just the initial value x^0 is given. By thus obtaining the quantities x_i^M $(i = 1, \ldots, n)$ and the function S itself as functions of the variables $u_1^0, \ldots, u_r^{M-1}$, one can determine the extrema of S with the conditions that $u^m \in U$ $(m = 0, \ldots, M-1)$ and $x^m \in G$. However, such a direct method of solving the problem in any complicated case turns out to be inadmissible in practice, due to the mass of computations required, even if a computer is available. Moreover, the necessity arises of providing a uniform method of solution valid for all problems of a given class and allowing the use of a uniform programing scheme. It is, therefore, necessary to seek methods which provide simpler (from the computational point of view) solutions of the problem posed. One of these methods is R. Bellman's dynamic programing method. Below, we present another method, related to an extension to discrete systems of L.S. Pontryagin's maximum principle.

In what follows, we shall consider systems which are linear in the variables $x_1, \ldots, x_n$, i.e., systems of the form

$$x_i^{m+1} - x^m = \tau\left[\sum_{k=1}^{n} a_{ik}^m x_k^m + \varphi_i(u_1^m, \ldots, u_r^m)\right] \quad (i = 1, \ldots, n), \tag{5.3}$$

where the functions φ_i are assumed to be continuous, and the coefficients a_{ik} can depend on the time m.

The limitation of our consideration to linear systems is explained by the fact that the extension of the maximum principle to discrete systems is possible, generally speaking, only in the linear case.*

We turn now to the formulation of the basic results. We consider n functions $p_1^m, \ldots, p_n^m$ of time m which satisfy the relationships

$$p_i^m - p_i^{m-1} = -\tau \sum_{1}^{m} p_s \frac{\partial f_s(x^m, u^m, m)}{\partial x_i} \quad (i = 1, \ldots, n), \tag{5.4}$$

where, by virtue of the linearity of the functions f_s,

$$\frac{\partial f_s(x^m, u^m, m)}{\partial x_i} = a_{si}^m. \tag{5.5}$$

We call the vector $p^m = (p_1^m, \ldots, p_n^m)$ the impulse.

By introducing the function

$$H(x, p, u, m) \equiv \tau \sum_{1}^{n} p_s f_s(x_1, \ldots, x_n;\ u_1, \ldots, u_r;\ m)$$

and denoting the corresponding first differences by the symbol Δ ($\Delta x_i^l \equiv x_i^{l+1} - x_i^l$, $\Delta p_i^l = p_i^{l+1} - p_i^l$, $l = 0, 1, 2, \ldots$), we can write relationships (5.3) and (5.4) in the form of the system

$$\Delta x_i^m = \frac{\partial H(x^m, p^m, u^m, m)}{\partial p_i^m}, \quad \Delta p_i^{m-1} = -\frac{\partial H(x^m, p^m, u^m, m)}{\partial x_i^m} \quad (i = 1, \ldots, n). \tag{5.6}$$

We now turn our attention to the circumstance that the values of the variables x^m, p^m and u^m at time m define the first differences of the coordinates at the same moment of time, but the first difference of the impulses at the previous moment of time. In other words, by knowing the values of the variables x^m, p^m and u^m at time m we can determine from (5.6) the coordinates at the following moment, $m + 1$, and the impulse at the previous moment, $m - 1$.

Let $u^m \in U$ be some admissible control, $x^m \equiv \alpha(m)$ the trajectory corresponding to it and $p^m \equiv \beta(m)$ some one of the vector functions of time which satisfy (5.6). By substituting in the function H the values of x^m and p^m, which are now definite functions of time, we obtain the quantity

$$K(m, u_1, \ldots, u_r) \equiv H[\alpha(m), \beta(m), u, m],$$

which, for each moment of time m, is a function of the point $u = (u_1, \ldots, u_r)$ which lies in set U of space $R(u_1, \ldots, u_r)$. We shall say that control u^m satisfies the maximum (minimum) condition with respect to vector function $p^m \equiv \beta(m)$ if, at each fixed moment of time m $(0 \le m \le M-1)$, the function $K(m,u)$ attains an absolute maximum (minimum) on the set U for values of the variables equal to the values of the controls at the same moment of time, i.e., for $u_k = u_k^m$.

*As $\tau \to 0$, the solution of the discrete problem tends, in a definite sense, to the solution of the corresponding continuous problem (cf. [6], where this assertion has been proven for linear systems and limitations of the form $|u_k| \le 1$). Therefore, for sufficiently small τ, the maximum principle turns out to be applicable in a sense.

Just as in the case of continuous systems, we first formulate the results for the problem with free trajectory right ends and then turn to the more general problems.

In the problem with free trajectory right ends, the set G occupies the entire space, which we symbolize by $G \sim X$. The following theorem establishes the discrete analogy to theorem 2 of section 2, part I.

Theorem 7. A necessary and sufficient condition for min-optimality (max-optimality) of control u^m ($m = 0, 1, \ldots, M-1$) in system (5.3) for $G \sim X$ is the holding, for u^m, of the maximum (minimum) condition with respect to the vector functions p^m which satisfy the relationships

$$p_i^{M-1} = -c_i \quad (i = 1, \ldots, n).$$

The proof of theorem 7 is completely analogous to the proof of theorem 2 for continuous systems, and is based on the following formula for the increment of value of function S with a variation of control u^m:

$$\delta S \equiv \sum_1^n c_i \delta x_i^M = -\sum_{m=0}^{M-1} [H(x^m, p^m, u^m + \delta u^m, m) - H(x^m, p^m, u^m, m)] - \eta, \tag{5.7}$$

where $\delta u^m \equiv (\delta u_1^m, \ldots, \delta u_r^M)$ are the increments of control and $\eta = \eta_1 + \eta_2$ are certain residual terms, where

$$\eta_1 = \frac{1}{2} \sum_{m=0}^{M-1} \sum_{s=1}^{n} \left[\frac{\partial H(y^m, u^m + \delta u^m, m)}{\partial y_s^m} - \frac{\partial H(y^m, u^m, m)}{\partial y_s^m} \right] \delta y_s^m,$$

$$\eta_2 = \frac{1}{2} \sum_{m=0}^{M-1} \sum_{s, q=1}^{n} \left[\frac{\partial^2 H(y^m + \vartheta_1^m \delta y^m, u^m + \delta u^m, m)}{\partial y_s^m \partial y_q^m} - \frac{\partial^2 H(y^m + \vartheta_2^m \delta y^m, u^m + \delta u^m, m)}{\partial y_s^m \partial y_q^m} \right] \delta y_s^m \delta y_q^m.$$

Here, $0 < \vartheta_1 < 1$, $0 < \vartheta_2 < 1$ and, for brevity, we introduced the vector $y = (y_1, \ldots, y_{2n})$ ($y_i = x_i$, $y_{n+i} = p_i$, $i = 1, \ldots, n$). For linear systems of the form of (5.3), the residual term η reduces to zero. The proof of formula (5.7) is carried out in complete analogy with the derivation of the corresponding formula for the increment of the functional's value.

We now turn to the more general problem wherein the "trajectory's" right end lies in some set $G \in X$. It is further assumed that set G is convex and closed.

We first formulate a sufficient condition for optimality. Just as in the analogous continuous problem (section 3f, part II), we shall denote by $b_i(y_1, \ldots, y_n)$ ($i = 1, \ldots, n$) the coefficients of the hyperplane bracketing set G at point $y = (y_1, \ldots, y_n)$. If $\underline{y}$ is an interior point, we then set $b_i(y_1, \ldots, y_n) = 0$. For each point $x(x_1, \ldots, x_n) \in G$ in the problem of minimizing function S we require that the inequality $\sum_1^n b_i(y_1, \ldots, y_n)(x_i - y_i) \leq 0$ hold, whereas we require that the inequality hold with reversed sense in the problem where the function is to be maximized.

Just as in the continuous case, we shall consider nondegenerate problems (section 3a, part II). In a nondegenerate problem, the set $G^* \in G$, on which the function $\phi(x) \equiv \sum_1^n c_i x_i$ assumes its least (greatest) value in comparison with the values for all other points $x \in G$, is not attainable from point x^0 within time M for the controls $u^m \in U$.

Now, if $x^0, x^1, \ldots, x^M$ is an optimal "trajectory," corresponding to control $u^0, u^1, \ldots, u^{M-1}$, the following theorem, establishing a sufficient condition for optimality, is valid.

Theorem 8. If, in system (5.3), control u^m ($m = 0, 1, \ldots, M-1$) satisfies the maximum (minimum) condition with respect to vector p^m, whose coordinates at time $m = M-1$ assume the values

$$p_i^{M-1} = -\lambda c_i - \mu b_i (x_1^M, \ldots, x_n^M) \quad (i = 1, \ldots, n),$$

where μ is nonnegative and λ is positive, then control u^m is min-optimal (max-optimal) with respect to $S = \sum_1^n c_i x_i^M$.

The proof of theorem 8 is based on theorem 7 and is carried out in complete analogy with the proof of theorem 4.

In the formulation of necessary conditions for optimality, it is necessary that one further requirement hold, this requirement amounting to the following. We consider the mapping of r-dimensional space $R(u_1, \ldots, u_r)$ on n-dimensional space $X(v_1, \ldots, v_n)$, given by the functions

$$v_i = \varphi_i (u_1, \ldots, u_r) \quad (i = 1, \ldots, n),$$

which define the right members of Eq. (5.3). We denote by V the set of all points of space X which are images of points which lie in set U of space R. We shall require that set V be convex (which occurs, in particular, if $v_i = b_i u$ and the variable $\underline{u}$ is of bounded modulus).

The following theorem establishes a necessary condition for optimality.

Theorem 9. Let set V be convex. Then, if control $u^m \in U$ ($m = 0, 1, \ldots, M-1$) in the nondegenerate problem is min-optimal (max-optimal) with respect to $S = \sum_1^n c_i x_i^M$, there then exist vector functions p^m with respect to which control u^m satisfies the maximum (minimum) condition, where the coordinates of vectors p^m at time $m = M-1$ assume the values

$$p_i^{M-1} = -\lambda c_i - \mu b_i (x_1^M, \ldots, x_n^M) \quad (i = 1, \ldots, n),$$

where the constants λ and μ are nonnegative and do not vanish simultaneously.

The proof of theorem 9 is obtained in the following way (for example, for the case of min-optimality). By the conditions of the theorem, the set W_T of the points of space X which are attainable in time M with the control $u^m \in U$ is a convex set. The set G^-, which is the set of those points $x \in G$ for which $\sum_1^n c_i (x_i - x_i^M) \leqslant 0$, is also convex, where the points common to G^- and W_T (in particular, point x^M) can only be boundary points of the two sets. The foregoing defines the hyperplane A which separates sets G^- and W_T and which is a bracket for set W_T. By so choosing the signs of the coefficients a_i ($i = 1, \ldots, n$) of hyperplane A that the inequality $\sum_1^n a_i (x_i - x_i^M) \geqslant 0$, holds for any point $x \in W_T$, we can convince ourselves that control u^m is min-optimal for $S = \sum_1^n a_i x_i^M$ in the problem with free right ends and, thanks to theorem 7, satisfies the maximum condition with respect to the vector function p^m which, for $m = M-1$, assumes the values $p_i^{M-1} = -a_i$ ($i = 1, \ldots, n$). The relationships $a_i = \lambda c_i + \mu b_i (x_1^M, \ldots, x_n^M)$ are obtained exactly the same as in the continuous case (section 3, part II).

Theorems 7-9 are used for solving concrete problems by methods completely analogous to the method of using the maximum principle in continuous problems. Specifically, the maximum (minimum) principle allows one to express the control at each moment of time in terms of the coordinates and impulses of the system, thus providing the capability of eliminating the control from Eq. (5.6). The system of 2n finite difference equations thus obtained, with 2n variables and with boundary conditions stated in the formulation of the theorems, defines the optimal trajectory and impulses and, consequently, also defines the optimal control.

6. Use of the Maximum Principle for Solving Certain Problems Connected with the Dynamic Accuracy of Automatic Control Systems

The variational methods developed in optimal system theory have immediate application to the problems connected with the estimation of control processes. This is particularly the case when the problem of dynamic accuracy is so posed that the external stimuli on the system are assumed to be given only by a number of limitations: limitations on modulus, known values of the integral of the squared disturbances, etc. Indeed, the basic problem in the investigation of such cases amounts to choosing, from the class of admissible external stimuli, that one which is the most "dangerous" from the point of view of the accepted criterion of dynamic accuracy. Thus, for example, in B.V. Bulgakov's problem of the accumulation of deviations in linear systems [8-10], one chooses, from the class of modulus-limited external stimuli, those which lead to the maximum deviation of the controlled quantity. In [11, 12], the class of external stimuli is assumed to be given by definite values of the integral of the moduli, or the integral of the squared disturbance, and one then computes the maximum value which the integral of the squared controlled quantity might assume. It is obvious that, in all these cases, a variational problem is essentially solved. An explicit use of variational methods and, in particular, the methods developed in optimal system theory, holds out broad possibilities, both in the sense of the investigation of nonlinear systems and in the sense that the class of admissible external stimuli might be given in significantly more diverse ways.

B.V. Bulgakov's problem on the accumulation of deviations can be formulated in the following way, which is also meaningful for nonlinear systems. Let $u_1, \ldots, u_r$ be the external stimuli acting on the control system described by the differential equations

$$\dot{x}_i = f_i(x_1, \ldots, x_n, u_1, \ldots, u_r; t) \quad (i = 1, \ldots, n). \tag{6.1}$$

All that is known about an external stimulus $u = (u_1, \ldots, u_r)$ is that it satisfies some system of inequalities (in the particular case here, $|u_k| \leq a_k$, $k = 1, \ldots, r$). It is required to find the maximum possible deviation $x_1(T)$ which the controlled quantity x_1 can undergo at time $t = T$.

The problem just formulated is identical with the problem considered by us in section 2, part I, whose theorem 1 gives the capability of solving the problem of accumulation of deviations for nonlinear systems with diverse limitations on the external stimuli. The results already obtained for linear systems are easily obtained as particular cases here.* It turns out that the "impulse" vector considered in the maximum principle is intimately related to the impulsive response of linear systems.

For example, we now show that, by means of the maximum principle, the simplest problem of deviation accumulation is solved for a system with constant coefficients, described by an equation of the form

$$x^{(n)} + a_{n-1}x^{(n-1)} + \cdots + a_0 x = u, \; |u| \leqslant 1. \tag{6.2}$$

The relationship between the system's impulsive response and the "impulse" vector is particularly clear here. For simplicity of exposition, we shall write the formula for the case when $n = 2$,

$$\ddot{x} + a_1\dot{x} + a_0 x = u, \tag{6.3}$$

bearing in mind that the analogous relationships subsist in systems of higher order. We now write Eq. (6.3) in the form of the system

$$\dot{x}_0 = x_1, \; \dot{x}_1 = -a_0 x_0 - a_1 x_1 + u,$$

where $x = x_0$ is the coordinate to be controlled. The problem consists of finding the maximum of the function $S = x_0(T)$ for the fixed time T. We must, therefore, solve the problem with free trajectory right ends, whereby $c_0 = 1$ and $c_1 = 0$. In correspondence with the general methodology, we set up the function

*In those cases when the derivatives of the external stimuli enter into the system equations, we should use the results obtained in [4].

$$H = p_0x_1 - p_1(a_0x_0 + a_1x_1) + p_1u$$

and the system of equations

$$\dot{p}_0 = -\frac{\partial H}{\partial x_0} \equiv a_0p_1, \quad \dot{p}_1 = -\frac{\partial H}{\partial x_1} \equiv -p_0 + a_1p_1. \tag{6.4}$$

The boundary conditions are written in the form

$$p_0(T) = -c_0 \equiv -1, \quad p_1(T) = -c_1 \equiv 0. \tag{6.5}$$

Since we seek to maximize the functional, we use the minimum condition. The minimum of function H with respect to u is obviously attained at the boundary points: for $p_1 > 0$, $u = -1$ and, for $p_1 < 0$, $u = +1$, i.e.,

$$u(t) = -\operatorname{sign} p_1(t). \tag{6.6}$$

The alternation of the intervals on which the external stimulus assumes the values +1 and −1 is thus determined by the variations in time of the function $p_1(t)$. We now replace system (6.4) by one equation in $p \equiv \equiv p_1(t)$

$$\ddot{p} - a_1\dot{p} + a_0p = 0. \tag{6.7}$$

In (6.7) we carry out the change of variables, $W(t) = -p(T-t)$. Since $\dot{W}(t) = \dot{p}(T-t)$ and $\ddot{W} = \ddot{p}(T-t)$, (6.7) is written in the form of the following equation in the function $W(t)$:

$$\ddot{W}(t) + a_1\dot{W}(t) + a_0W(t) = 0 \tag{6.8}$$

with boundary conditions [by virtue of (6.5)]

$$W(0) = 0, \quad \dot{W}(0) = 1. \tag{6.9}$$

However, as is well known, the solution of Eq. (6.8) with initial data given by (6.9) gives the system's impulsive response. The left members of Eq. (6.8) and the sought-for Eq. (6.3) coincide. Thus, $W(t)$ is the impulsive response of control system (6.3), and component p_1 of the "impulse" vector (p_0, p_1) equals

$$p_1(t) = -W(T-t). \tag{6.10}$$

Since $\dot{p}_0 = a_0p_1$ and $p_0(T) = -1$, it easily follows that

$$p_0(t) = -1 + h(T-t), \tag{6.11}$$

where $h(t) = \int_0^t W(z)\,dz$, i.e., the system's reaction to a unit step function input. The most "dangerous" stimulus, by virtue of (6.6) and (6.10), is determined from the well-known formula

$$u(t) = \operatorname{sign} W(T-t). \tag{6.12}$$

Also easily solved are problems in which, in addition to the limitation $|u| \leq 1$, there are limitations of the type $\int_0^T |u|\,dt \leqslant A$ or $\int_0^T u^2\,dt \leqslant A$.* With such limitations, one may seek, not only the maximum deviation of the controlled quantity but also, for example, the maximum value of the integral of the square of this

*We note that, when there are "integral" limitations, we have to deal, generally speaking, with a problem in which the trajectory right ends are no longer free (cf. section 1, part I).

quantity (analogously to [12] where, however, $T = \infty$ and the limitation $|u| \leq 1$ is lacking). In linear cases, the solutions of such problems can be expressed in terms of the system's impulsive response.

The results of section 5 indicate how to expand the above-considered methods for investigating the dynamic accuracy for systems' control of discrete linear systems.

APPENDIX III

Proof of Theorem 6. For definiteness, we consider the case where a minimum of the functions $\sum_1^n c_i x_i(T)$. is sought. The theorem is proved in an analogous way when the case of maximizing the functional is concerned.

As before, we shall assume that the right members of the system

$$\dot{x}_i = f_i(x, u, t) \qquad (i = 1, \ldots, n, \; u(t) \in U) \tag{III.1}$$

are continuous in the set of arguments (x,u,t) and have continuous first and second partial derivatives with respect to the arguments $(x_1, \ldots, x_n)$.

Let (x^0, t_0) be some point in L, and let $u^* \equiv u^*(x^0, t_0, t)$ and $x^*(t) \equiv x^*(x^0, t_0, t)$ be, respectively, the optimal control and the optimal trajectory corresponding to a minimum of the functional $\sum_1^n c_i x_i(T)$ for a fixed value of T.

Further, at time $t = t'$ $(t_0 \leq t' \leq T)$, let the system be in state $x = x'$. On the segment $[t', T]$ we shall use the control $u^*(t)$. With this, the function $\sum_1^n c_i x_i(T)$ assumes the value $\Phi_T(x', t')$ which depends exclusively on (x', t') since the control $u^*(t)$ is given. By varying the state of the system (x', t'), we obtain the function

$$\Phi_T(x', t') \equiv \Phi_T[x', t', \{u^*(x^0, t_0, t)\}],$$

which has for its value at each fixed point $(x', t') \in L$ the value which the functional assumes on the control $u^*(t)$ $(t' \leq t \leq T)$ for the system's initial state being (x', t'). The symbol $\{u^*(x^0, t_0, t)\}$ (which will be omitted in the sequel) indicates that the function $\Phi_T(x', t')$ is meaningful only if the control $u^*(t)$ is previously given. Let $x(t) \equiv x(x', t', t)$ be the system's trajectory corresponding to control $u^*(t)$ with initial state (x', t'). By definition,

$$\Phi_T(x', t') \equiv \sum_1^n c_i x_i(x', t', T). \tag{III.2}$$

Since the functional to be minimized is defined by the final value only (at time $t = T$) of the system's coordinate, it is then obvious that

$$\Phi_T[x(t), t] = \Phi_T(x', t') \qquad (t' \leqslant t \leqslant T), \tag{III.3}$$

i.e., the values of the functional on the control $u^*(t)$ for initial states lying on the trajectory $x(t) \equiv x(x', t', t)$ are all equal to one another.

It follows, from our assumptions as to the right member of Eqs. (III.1) and from the piecewise continuity of control $u^*(t)$, that the solution $x = x(x', t', t)$ of system (1) is continuous in the set of initial data (x', t') and has all continuous first-order partial derivatives and continuous second-order partial derivatives $\partial^2 x_i / \partial x'_s \partial x'_q$, $\partial^2 x_i / \partial x'_s \partial t'$ $(i, s, q = 1, \ldots, n)$ (cf. [7]).*

*In [7], the proof of the theorem on the continuity and differentiability of the solution with respect to the initial data is carried out on the assumption that the right members of the differential equations are continuous in t. By assuming that, at the points of discontinuity, $u^*(t) = u^*(t+0)$ and by using the so-called theorem on half-interval continuity, one can easily show that the solution of system (1) of equations is differentiable the proper number of times with respect to the initial data even in the case considered, when control $u^*(t)$ is piecewise continuous and has a finite number of first-order discontinuities.

From this follow the continuity and the existence of the corresponding partial derivatives of the function $\Phi_T(x', t') \equiv \sum_1^n c_i x_i(x', t', T)$. By taking this into account, and by then taking the total derivative of Eq. (III.3) with respect to t, we get, by virtue of (III.1),

$$\frac{\partial \Phi_T[x(x', t', t), t]}{\partial t} = -\sum_1^n \frac{\partial \Phi_T[x(x', t', t), t]}{\partial x_s} f_s[x(x', t', t), u^*(t), t].$$

This last relationship holds identically for any x', t' and $t(t' \le t \le T)$. Therefore, by setting $t = t'$, and keeping in mind that $x(x', t', t) = x'$ and then discarding the primes, we obtain the identity

$$\frac{\partial \Phi_T(x, t)}{\partial t} = -\sum_1^n \frac{\partial \Phi_T(x, t)}{\partial x_s} f_s[x, u^*(t), t]. \tag{III.4}$$

We now consider the functions

$$p_i(t) \equiv -\frac{\partial \Phi_T[x^*(t), t]}{\partial x_i} \qquad (i = 1, \ldots, n). \tag{III.5}$$

We first note that, by virtue of the well-known relationships

$$\frac{\partial x_i(x^0, T, T)}{\partial x_j^0} = \begin{cases} 1 & i = j, \\ 0 & i \neq j \end{cases}$$

(cf. for example, [7]), we have the equality

$$\frac{\partial \Phi_T(x^0, T)}{\partial x_i^0} \equiv \sum_1^n c_s \frac{\partial x_s(x^0, T, T)}{\partial x_i^0} = c_i \qquad (i = 1, \ldots, n)$$

and, consequently,

$$p_i(T) = -c_i. \tag{III.6}$$

We now derive the system of differential equations which are satisfied by the functions $p_i(t)$. We differentiate (III.5) with respect to t (the corresponding derivatives exist and are continuous):

$$\dot{p}_i(t) = -\frac{\partial^2 \Phi_T[x^*(t), t]}{\partial x_i \partial t} - \sum_{s=1}^n \frac{\partial^2 \Phi_T[x^*(t), t]}{\partial x_i \partial x_s} \dot{x}_s^*(t). \tag{III.7}$$

From (III.1)

$$\dot{x}_s^* = f_s[x^*(t), u^*(t), t] \qquad (s = 1, \ldots, n). \tag{III.8}$$

By differentiating (III.4) with respect to x_i $(i = 1, \ldots, n)$, we get

$$\frac{\partial^2 \Phi_T(x, t)}{\partial x_i \partial t} = -\sum_1^n \frac{\partial^2 \Phi_T(x, t)}{\partial x_i \partial x_s} f_s[x, u^*(t), t] - \\ -\sum_1^n \frac{\partial \Phi_T(x, t)}{\partial x_s} \frac{\partial f_s[x, u^*(t), t]}{\partial x_i}. \tag{III.9}$$

Now, by substituting $x = x^*(t)$ in (III.9), by taking (III.5) into account and by substituting (III.8) and (III.9) into (III.7), we obtain the system of differential equations sought:

$$\dot{p}_i(t) = -\sum_1^n p_s(t) \frac{\partial f_s[x^*(t), u^*(t), t]}{\partial x_i} \qquad (i = 1, \ldots, n). \tag{III.10}$$

But, according to theorem 1 of part I, optimal control $u^*(t)$ satisfies the maximum condition with respect to vector $p(t)$, defined by relationships (III.5) which are subject to system (III.10) of equations and conditions (III.6). Therefore, by substituting $x = x^*(t)$ in (III.4) and by noting that the right member of (III.4) is the expression

$$H[x^*(t), p(t), u^*(t), t] \equiv \sum_1^n p_s(t) f_s[x^*(t), u^*(t), t],$$

we obtain

$$\frac{\partial \Phi_T[x^*(t), t]}{\partial t} = \max_{u \in U} \sum_1^n \left(\frac{\partial \Phi_T[x^*(t), t]}{\partial x_s} \right) f_s[x^*(t), u, t]. \tag{III.11}$$

In particular, for $t = t_0$, $x^*(t_0) = x^0$,

$$\frac{\partial \Phi_T(x^0, t_0)}{\partial t_0} = \max_{u^\epsilon U} \sum_1^n \left(-\frac{\partial \Phi_T(x^0, t_0)}{\partial x_s^0} \right) f_s(x^0, u^0, t_0). \tag{III.12}$$

We now establish the relationship between the functions $\Phi_T(x,t)$ and $S_T(x,t)$. By definition, for any point (x,t) in space L the following equation holds

$$\Phi_T(x, t) \geqslant S_T(x, t), \tag{III.13}$$

for which at points $\underline{x}$, $\underline{t}$ lying along the trajectory we always have:

$$\Phi_T(x, t) = S_T(x, t) \qquad (x = x^*(t)). \tag{III.14}$$

It follows from this that, in the $(n+2)$-dimensional space (z,x,t), the surfaces $z = \Phi_T(x,t)$ and $z = S_T(x,t)$ do not intersect, and have the common curve γ, whose projection on the subspace L gives the optimal trajectory $x = x^*(t)$. In other words, the surfaces $z = \Phi_T(x,t)$ and $z = S_T(x,t)$ are tangent in the curve γ. Now, if the function $S_T(x,t)$ is continuous and differentiable with respect to (x,t) in a neighborhood of the optimal trajectory $x = x^*(t)$ (the continuity and differentiability of the function $\Phi_T(x,t)$ were noted by us above), then the following relationships are valid everywhere on the optimal trajectory

$$\frac{\partial \Phi_T(x, t)}{\partial t} = \frac{\partial S_T(x, t)}{\partial t}, \quad \frac{\partial \Phi_T(x, t)}{\partial x_i} = \frac{\partial S_T(x, t)}{\partial x_i}$$

$$[i = 1, \ldots, n; \; x = x^*(t)]. \tag{III.15}$$

By assuming, in accordance with the conditions of the theorem that $(x^*(t), t)$ Γ, and by thus guaranteeing that (III.15) holds, we obtain, in correspondence with definition (III.5),

$$p_i(t) = -\frac{\partial S_T[x^*(t), t]}{\partial x_i} \qquad (i = 1, \ldots, n). \tag{III.16}$$

By replacing $\partial\Phi_T/\partial x_i$ and $\partial\Phi_T/\partial t$ in (III.11) by $\partial S_T/\partial x_i$ and $\partial S_T/\partial t$, and by using (III.16), we obtain

$$\frac{\partial S_T[x^*(t), t]}{\partial t} = H(x^*(t), p(t), u^*(t), t]. \qquad \text{(III.17)}$$

Since we have proven that the optimal control satisfies the maximum condition with respect to vector p(t), defined by relationships (III.16), point 1 of theorem 6 is completely proven.

To prove point 2 of the theorem, it suffices to replace the partial derivatives of function Φ_T in (III.12) by the analogous derivatives of the function S_T, in accordance with (III.15). If we note, with this, that the equation obtained remains valid for arbitrary x^0, t_0, and if we discard the superscript (and subscript) 0, we will obtain the identity*

$$\frac{\partial S_T(x, t)}{\partial t} = \max_{u \in U} \sum_1^n \left(-\frac{\partial S_T(x, t)}{\partial x_s}\right) f_s(x, u, t). \qquad \text{(III.18)}$$

The condition

$$S_T(x, T) = \sum_1^n c_s x_s \qquad \text{(III.19)}$$

is obvious in view of the definition of function $S_T(x,t)$, if one realizes that, by virtue of continuity, $\lim\limits_{t \to T} S_T(x, t) = S_T(x, T)$. Formulas (III.18) and (III.19) prove point 2. Thus, theorem 6 is completely proved.

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*In contradistinction to (III.18), one cannot discard the subscript 0 in (III.12) and consider the relationship thus obtained as a partial differential equation defining the function $\Phi_T(x,t)$. Equation (III.12) is not identically valid and, as is clear from (III.11), is valid only for points (x^0, t_0) lying on the optimal trajectory $x = x^*(t)$ which corresponds to some previously fixed optimal control $u^*(t)$.

A FREQUENCY METHOD FOR DETERMINING THE DYNAMIC CHARACTERISTICS OF OBJECTS OF AUTOMATIC CONTROL FROM DATA ON THEIR NORMAL USAGE

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A frequency method is presented for determining the dynamic characteristics of objects of automatic control from data obtained in the course of their normal usage, specifically, correlation and cross-correlation functions.

The paper considers linear objects with many inputs and outputs and with constant lags, and also multiloop linear systems in the presence of noise.

It is assumed for the analysis that the random processes occurring in the systems are ergodic and stationary, and that the objects under investigation are stable.

1. Present State of the Question and the Posing of the Problem

As is well known, the ordinary methods of determining the dynamic characteristics of industrial objects by measuring their reactions to artificial stimuli of definite forms applied at their inputs [1, 2] are inapplicable in many cases, for the following reasons.

1. It is undesirable or impossible to apply a disturbing stimulus of a special form at the object's input because this leads to a disruption of the normal course of the processes in the object.

2. Very frequently, random uncontrolled disturbances are superimposed on these stimuli. As a result, it turns out to be impossible to determine accurately the dynamic characteristics by means of standard input signals.

In view of these circumstances, great importance has recently been attached to a statistical method which permits the dynamic characteristics — transfer function or impulsive response — to be determined from correlation functions which are obtained from data on the normal usage of the objects. The present work is devoted to the theory of this method and to the methodology of its use. Works [3-6] present methods for machine solution of the Fredholm integral equation of the first type which relates the correlation function and the impulsive response. This equation, which is the starting point in the obtaining, by the statistical method, of the characteristics of objects with one input and output, is written in the following way:

$$R_{yx}(\tau) = \int_0^\infty R_x(\tau - \vartheta)\, k(\vartheta)\, d\vartheta, \tag{1}$$

where

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t+\tau)\, x(t)\, dt \tag{2}$$

is the correlation (autocorrelation) function of the process x(t), x(t) is an ergodic and stationary random process at the object's input,

$$R_{yx}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T y(t+\tau)\, x(t)\, dt \tag{3}$$

is the cross-correlation function of the processes x(t) and y(t), y(t) is the ergodic, stationary random process at the object's output.

The sought-for impulsive response k(t) must satisfy the condition of physical realizability

$$k(t) = 0 \text{ for } t < 0. \tag{4}$$

In [7], using a linear presentation of the problem, solutions are given for determining the dynamic characteristics of objects with several cross-correlated inputs and outputs. Two methods of solution are presented. The first method is based on the separation from each input of the component which is not correlated with the remaining input stimuli. In the second method, the problem is reduced to the choice of stimuli which are correlated with only one or, in the extreme case, with a small number of stimuli at the object's input.

In [8], this latter method is used for the analysis of single-loop and multiloop systems with noise applied inside the objects under investigation. Finally, in [4], there is considered an approximate method of determining the characteristics of objects closed by feedback paths and with internal noise present.

In all these works [3-8] solutions are carried out principally in the time domain, i.e., the determination of the objects' characteristics is reduced to the finding of the impulsive responses. This, in many cases, leads to horrendous computations, particularly when integral equations must be solved. It turns out that the problem can be significantly simplified if, by using the Fourier transform (cf., for example, [9]), one seeks the solution in the frequency domain, i.e., if one initially determines the objects' frequency characteristics, from which one can always go to the impulsive responses.

It is true that with this it is necessary to compute the spectral density from previously computed correlation functions. On the other hand, Eq. (1) for a stable object is very easily solved by dividing the spectral densities:

$$\Phi(i\omega) = \frac{S_{yx}(\omega)}{S_x(\omega)} \tag{5}$$

where $\Phi(i\omega)$ is the object's transfer function,

$$S_{yx}(\omega) = \int_{-\infty}^{+\infty} R_{yx}(\tau)\, e^{-i\omega\tau}\, d\tau, \tag{6}$$

$$S_x(\omega) = \int_{-\infty}^{+\infty} R_x(\tau)\, e^{-i\omega\tau}\, d\tau. \tag{7}$$

Solution (5) of Eq. (1) is given in [8-10]. In the first work it is given without proof, and in the two latter works it is obtained formally by means of the Fourier integral without the condition of physical realizability (4) being verified.

The task of the present paper is, first, to show that solution (5) is valid only for stable linear objects and, second, to apply the frequency method, based on the Fourier transform, to the investigation of a broad class of objects (with many inputs and outputs and with constant lags), as well as to the analysis of multiloop systems with internal noise present.

The solution of the first problem, with the detailed computations, is given in the Appendix to this paper, where we consider a stable linear object with one input and output and with constant lags. We, therefore, turn immediately to the consideration of the general case.

2. The Frequency Method of Determining the Dynamic Characteristics of Linear Objects with Many Inputs and Outputs and Constant Lags

As is well known, to obtain the dynamic characteristics of an object with n inputs and m outputs, it is necessary to find nm impulsive responses or transfer functions [7]. It is easily shown that, to solve this problem, it suffices to consider an object with n inputs and one output.

Indeed, by considering each output independently of the others, and by determining m times the dynamic characteristics of an object with n inputs and one output, we can successively determine all nm impulsive responses.

Thus, in the general case, the problem reduces to the determination of the dynamic characteristics of objects with n inputs and one output. To solve this problem, we make the following assumptions:

a) the random processes at the objects' inputs and outputs are ergodic and stationary;

b) the objects under investigation are stable;

c) the autocorrelation and cross-correlation functions of the processes at the input and output of an object are known.

In addition, for greater generality, we shall assume that the object possesses constant lags with respect to each input, and that the lags with respect to different inputs are not equal to each other.

We consider two methods of solving this problem.

A. Reduction of the Problem to the Solution of a System of Linear Algebraic Equations

For an object with n inputs and one output, the output signal $z(t)$, by virtue of the principle of superposition, may be written in the form

$$z(t)=\int_0^\infty x_1(t-\vartheta)\,k_1(\vartheta)\,d\vartheta+\ldots+\int_0^\infty x_n(t-\vartheta)\,k_n(\vartheta)\,d\vartheta, \tag{8}$$

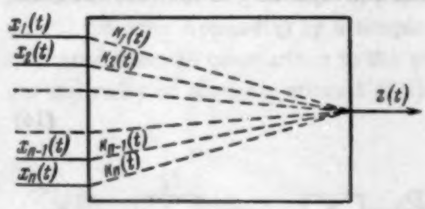


Fig. 1.

where $x_1(t), \ldots, x_n(t)$ are the disturbances at the inputs (Fig. 1), $k_1(t), \ldots, k_n(t)$ are the impulsive responses.

If we multiply both sides of Eq. (8), successively, by $x_1(t-\tau)/T, \ldots, x_n(t-\tau)/T$ and integrate with respect to t we obtain, by virtue of relationships (2) and (3), a system of n linear integral equations relating the n impulsive responses:

$$\begin{aligned}
R_{zx_1}(\tau)&=\int_{t_{d1}}^\infty R_{x_1}(\tau-\vartheta)\,k_1(\vartheta)\,d\vartheta+\ldots+\int_{t_{dn}}^\infty R_{x_nx_1}(\tau-\vartheta)\,k_n(\vartheta)\,d\vartheta,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
R_{zx_n}(\tau)&=\int_{t_{d1}}^\infty R_{x_1x_n}(\tau-\vartheta)\,k_1(\vartheta)\,d\vartheta+\ldots+\int_{t_{dn}}^\infty R_{x_n}(\tau-\vartheta)\,k_n(\vartheta)\,d\vartheta,
\end{aligned} \tag{9}$$

where $t_{d_1}, t_{d_2}, \ldots, t_{dn}$ are the constant lags with respect to the first, second, etc., inputs respectively.

In the particular case when $x_1(t), \ldots, x_n(t)$ are statistically independent or, in other words, are uncorrelated, i.e., when

$$R_{x_ix_j}(\tau)=0 \tag{10}$$

for all τ and for $i \neq j$, we obtain from system (9)

$$R_{zx_1}(\tau) = \int\limits_{t_{d1}}^{\infty} R_{x_1}(\tau - \vartheta)\, k_1(\vartheta)\, d\vartheta,$$

$$\cdots\cdots\cdots\cdots \tag{11}$$

$$R_{zx_n}(\tau) = \int\limits_{t_{dn}}^{\infty} R_{x_n}(\tau - \vartheta)\, k_n(\vartheta)\, d\vartheta.$$

Equations (11) are solved separately by the method presented in the Appendix. We, therefore, consider only the case when $x_1(t), \ldots, x_n(t)$ are cross-correlated. Thus, we will seek a solution of system (9), $k_1(t), \ldots, k_n(t)$, which satisfies the condition of physical realizability:

$$\begin{aligned} k_1(t) &= 0 \qquad (t < t_{d1}), \\ k_2(t) &= 0 \qquad (t < t_{d2}), \\ &\cdots\cdots \\ k_n(t) &= 0 \qquad (t < t_{dn}), \end{aligned} \tag{12}$$

where the constant lags, $t_{d_1}, t_{d_2}, \ldots, t_{dn}$ are not known beforehand.

We first find the solution of system (9) in the frequency domain. For this, by carrying out the computations analogous to those in the Appendix, we present system (9) in the frequency domain in the following way:

$$\begin{aligned} S_{zx_1}(\omega) &= S_{x_1}(\omega)\,\overline{\Phi_1(i\omega)} + \ldots + S_{x_nx_1}(\omega)\,\overline{\Phi_n(i\omega)}, \\ &\cdots\cdots\cdots\cdots \\ S_{zx_n}(\omega) &= S_{x_1x_n}(\omega)\,\overline{\Phi_1(i\omega)} + \ldots + S_{x_n}(\omega)\,\overline{\Phi_n(i\omega)}, \end{aligned} \tag{13}$$

where $\Phi_1(i\omega), \ldots, \Phi_n(i\omega)$ are the transfer functions corresponding to impulsive responses $k_1(t), \ldots, k_n(t)$, $\Phi_j(i\omega) = \Phi_j(i\omega)e^{-i\omega t_{dj}}$ $(j = 1, 2, \ldots, n)$ and $\Phi_j(i\omega)$ is the transfer function corresponding to the idealized link without lags,

$$S_{zx_i}(\omega) = \int\limits_{-\infty}^{+\infty} e^{-i\omega\tau} R_{zx_i}(\tau)\, d\tau, \tag{14}$$

$$S_{x_ix_k}(\omega) = \int\limits_{-\infty}^{+\infty} e^{-i\omega\tau} R_{x_ix_k}(\tau)\, d\tau, \tag{15}$$

$$S_{x_i}(\omega) = \int\limits_{-\infty}^{+\infty} e^{-i\omega\tau} R_{x_i}(\tau)\, d\tau. \tag{16}$$

By solving system (13) with respect to $\Phi_1(i\omega), \ldots, \Phi_n(i\omega)$, we find the object's transfer functions:

$$\overline{\Phi_j(i\omega)} = \frac{\sum\limits_{k=1}^{n} (-1)^{k+j} S_{zx_k}(\omega)\,\Delta_{kj}}{\Delta}, \tag{17}$$

where

$$\Delta = \begin{vmatrix} S_{x_1}(\omega) & \ldots & S_{x_nx_1}(\omega) \\ \cdots & \cdots & \cdots \\ S_{x_1x_n}(\omega) & \ldots & S_{x_n}(\omega) \end{vmatrix} \tag{18}$$

is the determinant of system (13) and Δ_{kj} is the minor obtained from the determinant by striking out its k-th row and j-th column.

By then applying the Fourier transform, we obtain the solution in the time domain as well:

$$k_j(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{\Phi_j(i\omega)}\, e^{i\omega t} d\omega \qquad (j = 1, 2, \ldots, n). \tag{19}$$

By repeating the argument given in the Appendix, we easily see that the solution found satisfies condition (12) for physical realizability. The method presented requires significantly less work than the solution in the time domain based on the separation of the component of each input which is uncorrelated with the remaining input stimuli.

We turn now to the consideration of still another method of obtaining the dynamic characteristics of an object with <u>n</u> inputs and one output.

B. Determination of Objects' Dynamic Characteristics by the Method of Selecting Uncorrelated Stimuli

As was shown earlier, for uncorrelated stimuli $x_1(t), \ldots, x_n(t)$, the dynamic characteristics of the object are determined independently of the remaining $n-1$ inputs. It can be shown, however, that in the general case when the stimuli $x_1(t), \ldots, x_n(t)$ are cross-correlated, the problem of determining the dynamic characteristics can be reduced to the solution of equations analogous to Eq. (1).

Indeed, let us have succeeded in choosing stimulus e_i which is correlated with the i-th input and uncorrelated with the remaining $n-1$ inputs. Then, multiplying Eq. (8) by $e_i(t-\tau)$ and integrating over <u>t</u>, we get, by virtue of relationship (3), that

$$R_{ze_i}(\tau) = \int_0^{\infty} R_{x_i e_i}(\tau - \vartheta)\, k_i(\vartheta)\, d\vartheta \qquad (i = 1, 2, \ldots, n). \tag{20}$$

This equation differs from Eq. (1) only in that the cross-correlation function $R_{x_i e_i}(\tau)$ appears in the integrand. Therefore, the solution of this equation in the frequency domain can be obtained analogously to that of Eq. (1) (cf. the Appendix) by a simple division of spectral densities. However, despite this simplicity, the method presented here, in comparison to the previous one, possesses an essential disadvantage. This disadvantage lies in the difficulty of choosing stimuli $e_i(t)$ which are not correlated with the $x_j(t)$ $(i \neq j)$ and with the lack of assurance that they are, in fact, uncorrelated. In the conclusion, we shall consider a case when, from the physical conditions, one can comparatively simply choose the stimuli $e_i(t)$ and, consequently, apply the method just described.

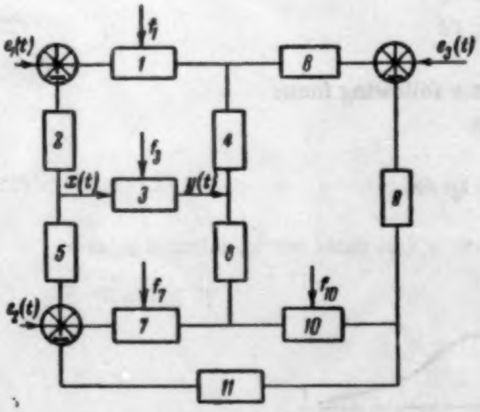


Fig. 2.

3. Statistical Analysis of Multiloop Control Systems with Internal Noise

With the system shown in Fig. 2 as an example, we shall demonstrate the application of the method of choosing uncorrelated stimuli to the analysis of multiloop control systems with internal noise. By assuming that the three assumptions of the previous section hold, we find, for example, the dynamic characteristics of link 3. As is easily seen, the presence of feedback in the system leads to the result that the stimulus at the link's input, $x(t)$, is correlated with the internal noise $f_3(t)$, and relationship (1) does not hold. Due to this, it is advantageous, in determining the dynamic characteristics of the object, to use the method of choosing stimuli which are uncorrelated with the internal noise. Indeed, let us, in addition to writing the process $x(t)$ and $y(t)$ at the object's input and output, write the process $E(t)$,

which is correlated with the input and, consequently, with the output stimuli, but is uncorrelated with the disturbances inside the object. Then, the object's dynamic characteristics are easily determined from the equation

$$R_{yE}(\tau) = \int_0^{\infty} R_{xE}(\tau - \vartheta)\, k(\vartheta)\, d\vartheta, \tag{21}$$

considered in the previous section.

For the quantity E(t), which is uncorrelated with the noise $f(t)$, we can choose, based on the hypothesis of the independence of the internal noise and the external stimuli, any one of the stimuli $e_1(t)$, $e_2(t)$, or $e_3(t)$ (Fig. 2).

SUMMARY

The paper presented a frequency method for determining the dynamic characteristics of objects of automatic control from data obtained in their normal processes of use – autocorrelation and cross-correlation functions – applicable to a broad class of stable linear objects – those with many inputs and outputs and with constant lags. The application of the method presented was shown for the analysis of stable elements of linear multiloop automatic control systems with internal noise.

The results presented for the method of statistical linearization of nonlinearities can be easily extended to quasi-linear systems.

APPENDIX

We consider a stable linear object with one input and one output. For greater generality, we choose the case when the object has a constant lag t_d. Such an object can be presented in the form of two series-connected links: an ideal link without lags 1 and a lagged link 2 (Fig. 3). We understand by a lagged link an element of a control system in which the output quantity reproduces, without distortion, the variations in the input quantity but with some fixed delay t_d [11].

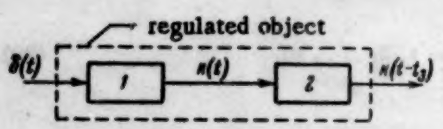


Fig. 3.

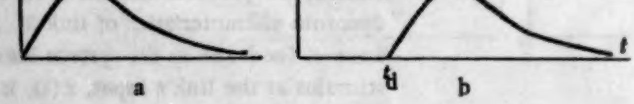
Thus, if k(t) for the ideal link without lag has the form shown in Fig. 4a, then the introduction of the additional lag link leads to a shift of k(t) to the right by the magnitude of the delay (Fig. 4b).

By virtue of this, the impulsive response for an object with a constant delay $k_d(t)$ can be given in the form

$$\begin{aligned} k_d(t) &= k(t - t_d), \qquad t > t_d \\ k_d(t) &= 0, \qquad t < t_d \end{aligned} \tag{22}$$

If expression (22) is taken into account, Eq. (1) is written in the following form:

$$R_{xy}(\tau) = \int_{t_d}^{\infty} R_x(\tau - \vartheta)\, k(\vartheta - t_d)\, d\vartheta. \tag{23}$$



a b

Fig. 4.

Thus, the determination of the dynamic characteristics of objects with constant lags from known correlation functions $R_x(\tau)$ and $R_{yx}(\tau)$ leads to the finding of the solution of Eq. (23) which satisfies the condition of physical realizability:

$$k(t-t_d)=k_d(t) = 0 \text{ for } t < t_d \tag{24}$$

With this, the constant lag t_d is assumed to be unknown beforehand. We now turn to the solution of Eq. (23). By multiplying both sides of Eq. (23) by $e^{-i\omega\tau}$ and integrating over τ from $-\infty$ to $+\infty$, we obtain

$$\int_{-\infty}^{+\infty} e^{-i\omega\tau} R_{yx}(\tau)\, d\tau = \int_{-\infty}^{+\infty} e^{-i\omega\tau} d\tau \int_{t_d}^{\infty} R_x(\tau-\vartheta)\, k(\vartheta - t_d)\, d\vartheta. \tag{25}$$

If, in the last integrals, we make the change of variable $\lambda = \tau - \vartheta$, we get

$$\int_{-\infty}^{+\infty} e^{-i\omega\tau} R_{yx}(\tau)\, d\tau = \int_{t_d}^{\infty} e^{-i\omega\vartheta} k(\vartheta - t_d)\, d\vartheta \int_{-\infty}^{+\infty} R_x(\lambda)\, e^{-i\omega\lambda} d\lambda. \tag{26}$$

By introducing the new variable $\varphi = \vartheta - t_d$, we transform Eq. (26) in the following way:

$$\int_{-\infty}^{+\infty} e^{-i\omega\tau} R_{yx}(\tau)\, d\tau = e^{-i\omega t_d} \int_{0}^{\infty} e^{-i\omega\varphi} k(\varphi)\, d\varphi \int_{-\infty}^{+\infty} R_x(\lambda)\, e^{-i\omega\lambda} d\lambda. \tag{27}$$

By using the spectral densities

$$S_{yx}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega\tau} R_{yx}(\tau)\, d\tau, \tag{28}$$

$$S_x(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega\tau} R_x(\tau)\, d\tau, \tag{29}$$

we rewrite relationship (27) in the form

$$S_{yx}(\omega) = e^{-i\omega t_d} \Phi(i\omega)\, S_x(\omega), \tag{30}$$

where

$$\Phi(i\omega) = \int_{0}^{\infty} e^{-i\omega\varphi} k(\varphi)\, d\varphi \tag{31}$$

is the transfer function of the ideal link without lags.

By denoting by

$$\overline{\Phi(i\omega)} = \Phi(i\omega)\, e^{-i\omega t_d} \tag{32}$$

the transfer function of the object with a constant lag, we have

$$\overline{\Phi(i\omega)} = \frac{S_{yx}(\omega)}{S_x(\omega)}. \tag{33}$$

If, as a particular case, we set $t_d = 0$ in expression (33), we obtain, for the ideal link without lags

$$\Phi(i\omega) = \frac{S_{yx}(\omega)}{S_x(\omega)}. \tag{34}$$

Thus, the real and imaginary frequency characteristics of a stable linear object are found by a simple division of the mutual spectral density $S_{yx}(\omega)$ by the spectral density $S_x(\omega)$.

If we again use the Fourier transform, we obtain the solution of Eq. (23) in the time domain:

$$k(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \overline{\Phi(i\omega)}\, e^{i\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{S_{yx}(\omega)}{S_x(\omega)} e^{i\omega t} d\omega. \tag{35}$$

The integral in (35) can be comparatively simply computed, either by the theory of residues, for which it is necessary to go from the frequency characteristic given in some frequency range to the analytical expression for the transfer function, or by a graphicoanalytic method [2].

Fig. 5.

In the second case, it is not necessary to find the analytical expression for the transfer function beforehand. It suffices to limit oneself to the graphic presentation of one real frequency characteristic.

We now show that the function k(t) just found satisfies condition (24). For this, we turn to the computation of k(t) for $t < t_d$, keeping in mind that, by virtue of the object's stability, all poles of the function lie in the left half-plane, i.e., that the function $\Phi(s)$ is analytic and bounded in the right half-plane.

By presenting the expression found for k(t) in the form

$$k(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Phi(i\omega)\, e^{i\omega(t-t_d)} d\omega, \tag{36}$$

we consider the integral

$$\frac{1}{2\pi i}\int_{C+} \Phi(s)\, e^{s(t-t_d)} ds, \tag{37}$$

along semicircle C^+ with radius $\underline{r}$, and lying in the right half-plane (Fig. 5).

For $t-t_d < 0$ and for $|r| \to \infty$, integral (20) tends to zero [12]. We may, therefore, write

$$k(t) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \Phi(s)\, e^{s(t-t_d)} ds = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} \Phi(s)\, e^{s(t-t_d)} ds + \frac{1}{2\pi i}\int_{C+} \Phi(s)\, e^{s(t-t_d)} ds \tag{38}$$

or

$$k(t) = \frac{1}{2\pi i}\oint \Phi(s)\, e^{s(t-t_d)} ds \quad \text{for} \quad t - t_d < 0, \tag{39}$$

where the integration is carried out along the closed contour consisting of the imaginary axis and the semicircle of infinite radius lying in the right half-plane.

According to the theory of residues, the value of this integral equals the sum of the residues of the integrand for all poles lying within the contour, multiplied by $2\pi i$. However, by virtue of the system's stability, function $\Phi(s)$ does not contain any poles in the right half-plane. Therefore,

$$k(t)=\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\Phi(s)\,e^{s(t-t_d)}ds=0 \quad \text{for} \quad t-t_d<0 \tag{40}$$

or

$$k(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{\Phi(i\omega)}\,e^{i\omega t}d\omega=0 \quad \text{for} \quad t-t_d<0. \tag{41}$$

Thus, for stable linear objects, the solution of Eq. (23), which satisfies condition (24) for physical realizability, is expressed by formulas (33) and (35). The constant lag t_d is determined after the impulsive response is computed (Fig. 4b) from the graph of $k_d(t)$.

In the proof just given, we assumed that $S_x(\omega)$ and $S_{yx}(\omega)$ were exact spectral densities. Then, by virtue of the relationship

$$\frac{S_{xy}(\omega)}{S_x(\omega)}=\lim_{T\to\infty}\frac{\frac{1}{T}Y_T(i\omega)X_T^*(i\omega)}{\frac{1}{T}X_T(i\omega)X_T^*(i\omega)}=\lim_{T\to\infty}\frac{Y_T(i\omega)}{X_T(i\omega)}, \tag{42}$$

where

$$X_T(i\omega)=\int_0^T x(t)\,e^{-i\omega t}dt, \qquad Y_T(i\omega)=\int_0^T y(t)\,e^{-i\omega t}dt,$$

the function $S_{yx}(\omega)/S_x(\omega)$ for a stable object will be analytic and bounded in the lower ω half-plane, since $X_T^*(i\omega)$ is factored out, and the corresponding singularities drop out.

In actuality, $S_x(\omega)$ and $S_{yx}(\omega)$ are computed with some error, and the zeros of $S_x(\omega)$ may not exactly coincide with the zeros of $S_{yx}(\omega)$ in the lower half-plane, i.e., $S_{yx}(\omega)/S_x(\omega)$ may have poles in the lower ω half-plane. This latter circumstance can be reflected in the results of computing $\overline{\Phi(i\omega)}$ by formula (33).

Indeed, in the case when the object under investigation has a very small margin of stability, at the points on the ω axis close to the poles of $S_{yx}(\omega)/S_x(\omega)$, the computed frequency characteristic may have sharp jumps. Such values of $\overline{\Phi(i\omega)}$ should be either rendered more exact by varying the path of integration in expression (25), or should be ignored completely.

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ON THE SYNTHESIS OF LINEAR AUTOMATIC CONTROL SYSTEMS WITH VARIABLE PARAMETERS

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A method is presented for determining, from the optimal weight function of an entire system and the weight functions of the known links in the system, the optimal weight function of the system's correcting link, which may be placed either on the forward path or in a feedback path. The method presented here allows one to circumvent the difficulties associated with the necessity of solving Volterra integral equations of the first type.

In designing an automatic control system which will operate under noisy conditions, one must know its optimal dynamic characteristics, i.e., those which will provide the greatest possible accuracy of system operation under the given conditions. The recently developed theory of optimal systems allows one to determine the optimal dynamic characteristics of an automatic control system if one knows the statistical characteristics of the input signal and noise. Once one knows the system's optimal dynamic characteristics, one can try so to choose the design schematic and the parameters that the dynamic characteristics thus obtained will be close to the optimal. There is still no general solution to this problem of automatic control system synthesis. In the present paper, the attempt is made to cite one of the possible ways of solving the problem of synthesizing a linear automatic control system if, as the system's optimal dynamic characteristic, its weight function $W(t, \tau)$ is given.

A designer is ordinarily given part of the design schematic and some of the parameters of the system to be designed, variations of which are either impossible or, for some reason or other, undesirable. In order to provide the required dynamic characteristics of the system in such a case, one can include in it some correcting links. It is possible to determine the optimal weight functions of the connecting links from the given optimal weight function of the system, found by means of optimal system theory, and from the weight functions of the given links of the system. Knowing the weight functions of the correcting links, one can find the differential equations by which they must be described [1, 3]. The differential equations which describe the operation of the correcting links completely determine their structure and parameters, which can be found for each concrete form of differential equation. The questions involved in designing the schematic for the correcting links from their differential equations are not considered in this paper.

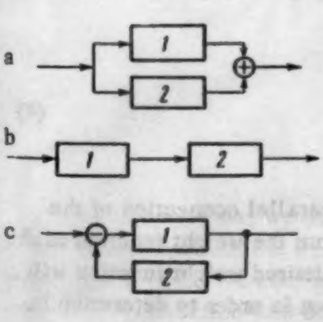

Fig. 1.

In order to determine the optimal weight functions of the correcting links, one must know the relationships between the weight functions of the individual links and their interconnections. Any automatic control system consists of three types of link interconnections: parallel (Fig. 1a), series (Fig. 1b), and with a feedback path (Fig. 1c). If we know, therefore, for each of these three types of connection, how to find the weight function of one link from the weight function of the entire connection and the weight function of the other link in the connection, then we can always find the weight function of a correcting link which enters into a system of any degree of complexity.

The relationships between the links' weight functions and the weight functions of their standard interconnections can be derived in the following way. For a parallel connection of the links we have, as an obvious consequence of the system's linearity,

$$W_{12}(t, \tau) = W_1(t, \tau) + W_2(t, \tau), \tag{1}$$

where the subscripts 1 and 2 denote the link to which the corresponding weight function appertains, and the subscript 12 denotes the connection of links 1 and 2 (Fig. 1).

By applying a delta-function signal to the input of series-connected links (Fig. 1b), we must obtain, at the output of link 1, the weight function $W_1(t, \tau)$ of this link and, at the output of link 2, the weight function $W_{12}(t, \tau)$ of the connection. If we now assume that $W_1(t, \tau)$ is the input signal to link 2, and take into account that, from the condition of physical realizability of the system,

$$W_1(t, \tau) \equiv 0 \text{ and } W_2(t, \tau) \equiv 0 \text{ for } t < \tau,$$

we obtain

$$W_{12}(t, \tau) = \int_\tau^t W_1(\tau_1, \tau) W_2(t, \tau_1)\, d\tau_1. \tag{2}$$

Finally, if we apply a delta-function signal to the input of a connection with feedback (Fig. 1c), we obtain the weight function $W_{12}(t, \tau)$ at the output. Then, in accordance with (2), at the output of link 2 we get

$\int_\tau^{\tau_1} W_{12}(\tau_2, \tau) W_2(\tau_1, \tau_2)\, d\tau_2$, at the input of link 1 we have a signal of the form

$\delta(\tau_1 - \tau) - \int_\tau^{\tau_1} W_{12}(\tau_2, \tau) W_2(\tau_1, \tau_2)\, d\tau_2$, and, at the output of link 1, we again have the weight function $W_{12}(t, \tau)$, i.e.,

$$W_{12}(t, \tau) = \int_\tau^t \left[\delta(\tau_1 - \tau) - \int_\tau^{\tau_1} W_{12}(\tau_2, \tau) W_2(\tau_1, \tau_2)\, d\tau_2\right] W_1(t, \tau_1)\, d\tau_1.$$

If we keep in mind that $\int_\tau^t \delta(\tau_1 - \tau) W_1(t, \tau_1)\, d\tau_1 = W_1(t, \tau)$, we obtain, finally,

$$W_{12}(t, \tau) = W_1(t, \tau) - \int_\tau^t W_1(t, \tau_1)\, d\tau_1 \int_\tau^{\tau_1} W_{12}(\tau_2, \tau) W_2(\tau_1, \tau_2)\, d\tau_2. \tag{3}$$

If there is an amplifying link with unit gain in the feedback path, then $W_2(\tau_1, \tau_2) = \delta(\tau_1 - \tau_2)$, and formula (3) takes the form:

$$W_{12}(t, \tau) = W_1(t, \tau) - \int_\tau^t W_1(t, \tau_1) W_{12}(\tau_1, \tau)\, d\tau_1. \tag{4}$$

It follows, from a consideration of (1), (2), and (3) that only in the case of a parallel connection of the links will the weight function of any link in this connection be easily determined from the weight function of the entire connection and the weight function of the other link. In all other cases, the desired weight function will enter into an integrand and, consequently, it is necessary to solve an integral equation in order to determine it.

If one is considering a connection with feedback, and it is necessary to find the weight function of link 1 (Fig. 1c) from the given weight function of the entire connection and the weight function of link 2 then, for its determination, it is necessary to solve, for $W_1(t,\tau)$, integral Eq. (3), which is a Volterra integral equation of the second type with kernel

$$\int_{\tau}^{\tau_1} W_{12}(\tau_2,\tau)\, W_2(\tau_1,\tau_2)\, d\tau_2.$$

The solution of a Volterra integral equation of the second type presents no serious difficulties, and may always be carried out by the method of successive approximations [2].

If the weight function of link 1 is given, and it is required to determine the weight function of link 2, which is in the feedback path, then it is necessary first to solve Eq. (3) for the integral $\int_{\tau}^{\tau_1} W_{12}(\tau_2,\tau) W_2(\tau_1,\tau_2) d\tau_2$, and then, knowing the value of this integral as a function of τ and τ_1, to determine $W_2(\tau_1,\tau_2)$. For this it is necessary both times to solve a Volterra integral equation of the first type. The problem of finding the weight function of either link in a series connection from the weight function of the other link and the weight function of the connection also leads to the solving of a Volterra integral equation of the first type. The solution of Volterra integral equations of the first type is a very arduous task which can be solved with sufficient speed and accuracy only by computers. In addition, Volterra integral equations of the first type may, in general, not have solutions.

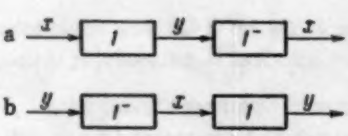

Fig. 2.

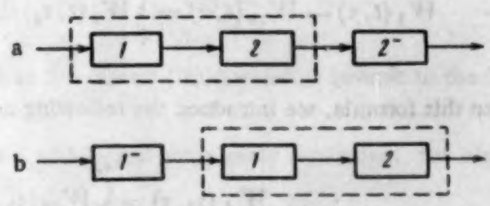

Fig. 3.

Let us attempt to avoid the difficulties connected with the solution of Volterra integral equations of the first type by using the concept of inverse links. Two links will be called inverses of each other if, when connected in series, they effect the identity transformation on any signal applied to the input of this connection, i.e., an inverse link carries out a transformation of its input signal which is inverse to the transformation effected by the direct link on its input signal (Fig. 2). A link which is inverse to a given link will be denoted by the same symbol, but with a superscript minus sign. It follows from Fig. 2 that it is possible to interchange inverse links which are series-connected. Since the weight function of the series connection of inverse links equals $\delta(t-\tau)$, the following relationships hold between the weight functions of mutually inverse links:

$$\int_{\tau}^{t} W(\tau_1,\tau)\, W^-(t,\tau_1)\, d\tau_1 = \delta(t-\tau), \tag{5}$$

$$\int_{\tau}^{t} W^-(\tau_1,\tau)\, W(t,\tau_1)\, d\tau_1 = \delta(t-\tau). \tag{6}$$

Suppose now that we have a series connection of links (Fig. 1b) and we know the weight function $W_{s}(t,\tau)$ of this connection and the weight function $W_2(t,\tau)$ of link 2. It is required to determine the weight function $W_1(t,\tau)$ of link 1. We add, to the given series connection, a link inverse to link 2 (Fig. 3a). The weight function of the connection thus obtained equals the weight function of link 1. On the basis of formula (2), we get

$$W_1(t,\tau)=\int_{\tau}^{t} W_{12}(\tau_1,\tau)\,W_2^-(t,\tau_1)\,d\tau_1. \quad (7)$$

Thus, the solution of a Volterra integral equation of the first type was reduced to a simple quadrature. The weight function $W_2^-(t,\tau)$ of the inverse link is easily determined from the differential equation which describes the operation of link 2. The differential equations of known links are ordinarily given.

If link 1 is known, and it is necessary to determine the weight function of link 2, then the given series connection should be connected to the output of a link inverse to link 1 (Fig. 3b). We then obtain

$$W_2(t,\tau)=\int_{\tau}^{t} W_1^-(\tau_1,\tau)\,W_{12}(t,\tau_1)\,d\tau_1. \quad (8)$$

Let it now be necessary for us to determine the weight function of link 2, located in the feedback path (Fig. 1c), for the given weight function of the entire connection and the weight function of link 1. We show now that, by using the concept of inverse links, we can reduce this problem to the solution of a Volterra integral equation of the second type. For this, we find the weight function of the link which is inverse to the entire configuration shown in Fig. 1c. We rewrite formula (3), which defines the weight function of a feedback connection, in the form

$$W_1(t,\tau)-W_{12}(t,\tau)=\int_{\tau}^{t} W_1(t,\tau_1)\,d\tau_1\int_{\tau}^{\tau_1} W_{12}(\tau_2,\tau)\,W_2(\tau_1,\tau_2)\,d\tau_2. \quad (9)$$

To analyze this formula, we introduce the following notation:

$$W_A(\tau_1,\tau)=\int_{\tau}^{\tau_1} W_{12}(\tau_2,\tau)\,W_2(\tau_1,\tau_2)\,d\tau_2, \quad (10)$$

$$W_B(t,\tau)=W_1(t,\tau)-W_{12}(t,\tau). \quad (11)$$

With this notation, formula (9) takes the following form:

$$W_B(t,\tau)=\int_{\tau}^{t} W_A(\tau_1,\tau)\,W_1(t,\tau_1)\,d\tau_1. \quad (12)$$

It follows from formula (12) that $W_B(t,\tau)$ is the weight function of a series connection of links with weight functions $W_A(t,\tau)$ and $W_1(t,\tau)$, respectively. In its turn, $W_A(t,\tau)$ is, according to (10), the weight function of a series connection of links with weight functions $W_{12}(t,\tau)$ and $W_2(t,\tau)$. Consequently, formula (9) expresses the fact that the weight function of a series connection of three links with weight functions $W_{12}(t,\tau)$, $W_2(t,\tau)$, and $W_1(t,\tau)$, respectively equals $W_B(t,\tau)=W_1(t,\tau)-W_{12}(t,\tau)$ (Fig. 4).

The weight function of link A, enclosed in dotted lines on Fig. 4, is defined by formula (10). On the other hand, it can be determined by the method of inverse links from the weight function of the entire configuration and the weight function of link 1:

$$\begin{aligned}W_A(t,\tau)&=\int_{\tau}^{t} W_B(\tau_1,\tau)W_1^-(t,\tau_1)\,d\tau_1=\int_{\tau}^{t}\Big[W_1(\tau_1,\tau)-W_{12}(\tau_1,\tau)\Big]W_1^-(t,\tau_1)\,d\tau_1=\\&=\delta(t-\tau)-\int_{\tau}^{t} W_{12}(\tau_1,\tau)\,W_1^-(t,\tau_1)\,d\tau_1.\end{aligned} \quad (13)$$

By comparing (10) and (13) we find, after some elementary transformations, that

$$\int_{\tau_1}^{t} W_{12}(\tau_1,\tau)\left[W_2(t,\tau_1)+W_1^-(t,\tau_1)\right]d\tau_1=\delta(t-\tau), \tag{14}$$

from whence, on the basis of (5), it follows that

$$W_{12}^-(t,\tau)=W_2(t,\tau)+W_1^-(t,\tau). \tag{15}$$

From this formula, $W_2(t,\tau)$ can be easily determined if the inverse weight function $W_{12}^-(t,\tau)$ is known or can be easily determined. In practice, it can be determined only in the rare cases when the differential equation describing the operation of the whole configuration is given or can be easily found. We now show how to determine weight function $W_2(t,\tau)$ if $W_{12}^-(t,\tau)$ is not known.

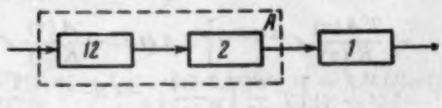

Fig. 4.

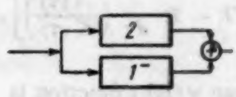

Fig. 5.

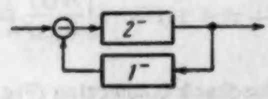

Fig. 6.

In accordance with (15), the block schematic of the circuit (link) which is inverse to the link shunted by a feedback path is represented as is shown in Fig. 5.

Since, in the block schematic obtained, links 2 and 1^- are completely equivalent, the block schematic of the direct link can be represented as shown in Fig. 6.

In the forward circuit of the feedback system just obtained, which is equivalent to the system shown in Fig. 1c, there is a link inverse to link 2. The weight function of this link is found from an equation of the form

$$W_{12}(t,\tau)=W_2^-(t,\tau)-\int_{\tau}^{t} W_2^-(t,\tau_1)d\tau_1\int_{\tau}^{\tau_1} W_{12}(\tau_2,\tau)\,W_1^-(\tau_1,\tau_2)\,d\tau_2; \tag{16}$$

which is a Volterra integral equation of the second type for $W_2^-(t,\tau)$.

The method just given allows one to determine the weight function of a link which is inverse to the link in the feedback path. For further solution of the synthesis problem, i.e., for finding the optimal weight functions of the simplest links, and also for finding the differential equation which describes the operation of the link, it is completely indifferent with which weight function one operates, whether with the direct or with the inverse.

It should also be mentioned that the method given here for finding the weight functions of links from given weight functions of their interconnections proves the existence of the solutions of Volterra integral equations of the first type of the form (2) or (3) for finding the weight function of a link in a feedback path.

EXAMPLES

1. Let there be given a series connection of links 1 and 2 whose weight function is

$$W_{12}(t,\tau)=1(t-\tau)A(\tau)e^{-\lambda(t-\tau)},$$

where link 1 is described by the differential equation

$$T\dot{x}+x=K(t)y.$$

We now determine the weight function of link 2.

The weight function of the link inverse to link 1 equals

$$W_1^-(t,\tau)=\frac{T}{K(t)}\,\delta'(t-\tau)+\frac{1}{K(t)}\,\delta(t-\tau).$$

From formula (8) we obtain

$$W_2(t,\tau)=\int_{\tau}^{t}\left[\frac{T}{K(\tau_1)}\,\delta'(\tau_1-\tau)+\frac{1}{K(\tau_1)}\,\delta(\tau_1-\tau)\right]1(t-\tau_1)\,A(\tau_1)\,e^{-\lambda(t-\tau_1)}\,d\tau_1=$$

$$=\int_{\tau}^{t}1(t-\tau_1)\frac{TA(\tau_1)}{K(\tau_1)}\,e^{-\lambda(t-\tau_1)}\,\delta'(\tau_1-\tau)\,d\tau_1+\int_{\tau}^{t}1(t-\tau_1)\frac{A(\tau_1)}{K(\tau_1)}\,e^{-\lambda(t-\tau_1)}\,d\tau_1=$$

$$=-\frac{d}{d\tau}\left[1(t-\tau)\frac{TA(\tau)}{K(\tau)}\,e^{-\lambda(t-\tau)}\right]+1(t-\tau)\frac{A(\tau)}{K(\tau)}\,e^{-\lambda(t-\tau)}=$$

$$=\delta(t-\tau)\frac{TA(\tau)}{K(\tau)}\,e^{-\lambda(t-\tau)}-1(t-\tau)\left[T\frac{d}{d\tau}\left(\frac{A(\tau)}{K(\tau)}\right)e^{-\lambda(t-\tau)}+\frac{TA(\tau)}{K(\tau)}\,\lambda\,e^{-\lambda(t-\tau)}\right]+$$

$$+1(t-\tau)\frac{A(\tau)}{K(\tau)}\,e^{-\lambda(t-\tau)}=$$

$$=\frac{TA(\tau)}{K(\tau)}\,\delta(t-\tau)+1(t-\tau)\left[\frac{A(\tau)}{K(\tau)}-\frac{A(\tau)}{K(\tau)}\,T\lambda-T\frac{d}{d\tau}\left(\frac{A(\tau)}{K(\tau)}\right)\right]e^{-\lambda(t-\tau)}.$$

2. We now consider a feedback connection (Fig. 1c) whose weight function is

$$W_{12}(t,\tau)=1(t-\tau)\,A(\tau)\,e^{-\lambda(t-\tau)},$$

where link 1 is described by the differential equation

$$T\dot{x}_1+x_1=K(t)\,y.$$

We determine the weight function of link 2.

From formula (15) we get

$$W_2(t,\tau)=W_{12}(t,\tau)-W_1^-(t,\tau),$$

where

$$W_1^-(t,\tau)=\frac{T}{K(t)}\,\delta'(t-\tau)+\frac{1}{K(t)}\,\delta(t-\tau).$$

In the given case, the differential equation which describes the operation of the connection as a whole is easily determined:

$$\frac{1}{\lambda}\,\dot{x}+x=\frac{A(t)}{\lambda}\,y.$$

Whence,

$$W_{12}^-(t,\tau)=\frac{1}{A(t)}\,\delta'(t-\tau)+\frac{\lambda}{A(t)}\,\delta(t-\tau).$$

Consequently,

$$W_2(t,\tau)=\left[\frac{1}{A(t)}-\frac{T}{K(t)}\right]\delta'(t-\tau)+\left[\frac{\lambda}{A(t)}-\frac{1}{K(t)}\right]\delta(t-\tau).$$

3. We now consider the feedback connection (Fig. 1c) whose weight function is

$$W_{12}(t,\tau) = 1(t-\tau)\,A(\tau)\exp\left(-\int_{\tau}^{t} B(t)\,dt\right),$$

and where link 1 is described by the differential equation

$$a_1(t)\dot{x}_1 + a_0(t)\,x_1 = y.$$

We determine weight function $W_2^-(t,\tau)$ without first determining $W_{12}^-(t,\tau)$. It is obvious that

$$W_1^-(t,\tau) = a_1(t)\,\delta'(t-\tau) + a_0(t)\,\delta(t-\tau).$$

From Eq. (16) we obtain

$$W_2^-(t,\tau) = 1(t-\tau)\,A(\tau)\exp\left(-\int_{\tau}^{t} B(t)\,dt\right) +$$
$$+\int_{\tau}^{t} W_2^-(t,\tau_1)\,d\tau_1 \int_{\tau}^{\tau_2} 1(\tau_2-\tau_1)\,A(\tau)\exp\left(-\int_{\tau}^{t} B(t)\,dt\right)[a_1(\tau_1)\,\delta'(\tau_1-\tau_2) +$$
$$+ a_0(\tau_1)\,\delta(\tau_1-\tau_2)]\,d\tau_2.$$

After some elementary transformations we get

$$W_2^-(t,\tau) = 1(t-\tau)\frac{A(\tau)}{1-A(\tau)\,a_1(\tau)}\exp\left(-\int_{\tau}^{t} B(t)\,dt\right) +$$
$$+\frac{A(\tau)}{1-A(\tau)\,a_1(\tau)}\int_{\tau}^{t} W_2^-(t,\tau_1)[a_0(\tau_1) - a_1(\tau_1)\,B(\tau_1)]\exp\left(-\int_{\tau}^{t} B(t)\,dt\right)d\tau_1.$$

This equation is solved by the method of successive approximations. As the first approximation we take

$$W_2^-(t,\tau) = 1(t-\tau)\frac{A(\tau)}{1-A(\tau)\,a_1(\tau)}\exp\left(-\int B(t)\,dt\right)$$

By substituting the first approximation $W_2^-(t,\tau)$ under the integral sign in the original integral equation we obtain the second approximation. The process of successive approximations continues until the last approximation does not differ appreciably from the previous one.

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SYNTHESIZING THE ELEMENTS OF LINEAR AUTOMATIC CONTROL SYSTEMS

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A method is proposed for synthesizing the elements of linear automatic control systems. The method is based on the use of time characteristics and relationships obtained from the system's (and links') integral equations with kernels in the form of polynomials, in conjunction with the D-decomposition of parameter space.

The method can be used, in particular, for programing the synthesis problem for computer solution.

For the construction of an automatic control system with given dynamic characteristics, it is required to determine the individual parameters of various links, and to find completely certain links of the system.

A number of engineering methods have been developed for synthesizing the elements of control systems [1-6], but it is impossible to consider the problem as having been completely solved.

The method proposed in the present work for synthesizing linear systems with constant parameters, this method being based on the use of time characteristics (transient responses) and Volterra integral equations with polynomial kernels in conjunction with the D-decomposition of parameter space, allows one to make broad use of computers.

1. Basic Relationships

One can, from the coordinate representation of a system, obtain an integral equation in this coordinate where the kernel is in the form of a polynomial [6]. Let the transient response in the system under investigation be known:

$$x(t) \doteqdot X(p) = \frac{b_m + b_{m-1}p + \cdots + b_1p^{m-1} + b_0p^m}{a_n + a_{n-1}p + \cdots + a_1p^{n-1} + a_0p^n}, \tag{1}$$

where $m \leq n$.

We divide both sides of the equation by p^n:

$$(a_n + a_{n-1}p + \cdots + a_0p^n)\frac{X(p)}{p^n} = \frac{1}{p^n}(b_m + b_{m-1}p + \cdots + b_0p^m). \tag{2}$$

We then introduce the notation

$$\frac{X(p)}{p^n} = Y(p), \tag{3}$$

which, when we transfer back to the time domain, gives

$$x(t) = y^{(n)}(t), \qquad y(t) = \int_0^t \int_0^t \cdots \int_0^t x(t)\,dt \ldots dt. \tag{4}$$

For the auxiliary function y(t), as follows from expression (4), there are zero initial conditions, $y_0 = y'_0 = \ldots = y_0^{(n-1)} = 0$, since for $t = 0$, the function y(t) and its $n-1$ derivatives vanish.

By substituting expression (3) in (2) and returning to the time domain, we obtain

$$a_n y(t) + a_{n-1} y'(t) + \cdots + a_0 y^{(n)}(t) = \\ = b_m \frac{t^n}{n!} + b_{m-1} \frac{t^{n-1}}{(n-1)!} + \cdots + b_0 \frac{t^{n-m}}{(n-m)!}. \tag{5}$$

We then write the Maclaurin expansion for function y(t)

$$y(t) = y_0 + y'_0 t + \cdots + \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} y^{(n)}(s)\,ds = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} x(s)\,ds. \tag{6}$$

If we differentiate Eq. (6) n times and substitute the expressions for y(t), y'(t), . . . , etc., thus obtained in Eq. (5), we obtain a Volterra integral equation of the second type in x(t):

$$a_0 x(t) - \int_0^t K(t, s)\, x(s)\,ds = N(t). \tag{7}$$

Here, the kernel is

$$K(t, s) = -\sum_{k=1}^{n} a_k \frac{(t-s)^{k-1}}{(k-1)!} \tag{8}$$

and the free term is

$$N(t) = \sum_{k=0}^{m} b_k \frac{t^{n-m+k}}{(n-m+k)!}. \tag{9}$$

The integral equation with the kernel given in (8) occurs in the literature [7]. There, however, the equation is in a higher derivative, and not in the coordinate x(t) itself, as is the case in the present paper.

The kernel in (8) and the free term in (9) of Eq. (7) are linear functions of the representation coefficients a_k and b_k.

The definite integral in expression (7) can be computed approximately by using one of the quadrature formulas. If we take d as the integration increment and then apply the trapezoid formula for calculating the integral, we obtain the well-known expression for the series of successive ordinates of curve x(t) [4].

For the first ordinate,

$$x_1 = \frac{N_1 + d\,\frac{x_0}{2} K_1}{a_0 + a_1 \frac{d}{2}}. \tag{10}$$

For any moment of time $t_v = vd$,

$$x_v = \frac{N_v + d\left(\frac{x_0}{2}K_v + x_1K_{v-1} + \ldots + x_{v-1}K_1\right)}{a_0 + a_1\frac{d}{2}}, \tag{11}$$

where

$$x_v = x(t_v),\ N_v = N(t_v)\ \text{и}\ K_v = -\sum_{k=1}^{n} a_k \frac{t_v^{k-1}}{(k-1)!}. \tag{12}$$

The error of this method is determined by the error introduced by the trapezoid formula in expression (11). In order to decrease this error one can, instead of the trapezoid formula, use Laplace's quadrature formula.

Equations (7) and (11) establish linear relationships between the coefficients of the numerator and denominator of representation (1) and the particular values of the coordinate $x(t)$, and are used below for the purpose of synthesis. For given numerical values of a_k and b_k, the same expressions (and somewhat transformed expressions) can be used for finding successively particular values of coordinate $x(t)$ (cf. [6]).

2. Determining the System's Parameters

Initially, we consider the synthesis problem as thus posed: the structural scheme of the system is given; defined are several parameters of one or several of the system's links (gains, time constants, etc.). Let the representation of coordinate $x(t)$ be given by expression (1), where a_k and b_k depend linearly on the system parameters to be determined.

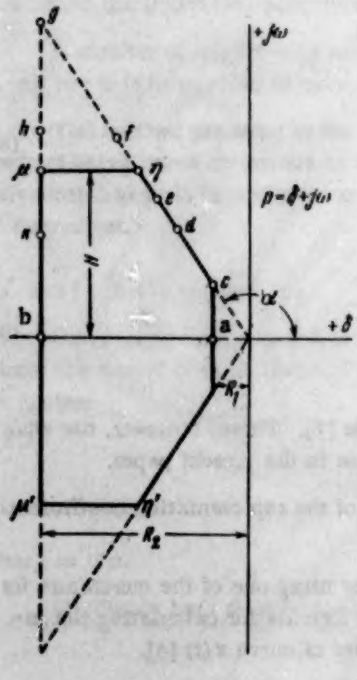

Fig. 1.

The parameters must be determined so as to provide:

1) a given degree of stability and oscillation of the system;

2) an accepted form of the transient response curve for the coordinate $x(t)$, a minimum and maximum value of the transient response, a given velocity and duration of the transient response.

Point 1 is attained by introducing into the calculations the conditions which determine the placement of the roots of the characteristic equation inside the given region of the complex p plane $ac\eta\mu b\mu'\eta'a$ (Fig. 1), called the domain of given quality in the sequel, for which one uses, as suggested by Yu.I. Neimark, the D-decomposition of the plane of one or two parameters [5].

Point 2 is attained by moving the points of the actual transient response curve of $x(t)$ close to the given curve $x^0(t)$, for which one uses expressions (9), (11), and (12).

Several parameters being sought may be determined or eliminated from the aforementioned equations by using the well-known relationships of the operational method, providing coincidence of the actual process $x(t)$ with the given $x^0(t)$ for $t = 0$ and $t = \infty$:

$$x(0) = x^0(0) = X(\infty) = \frac{b_0}{a_0},\quad x(\infty) = x^0(\infty) = X(0) = \frac{b_n}{a_n}. \tag{13}$$

In accordance with the methodology being considered here, the remaining q parameters of the system are determined in the following way.

1. We set up $q-2$ (or $q-1$) equations from formula (11) for $q-2$ $(q-1)$ points lying on the curve of the given process (Fig. 2):

$$x_\nu \left(a_0 + a_1 \frac{d}{2}\right) = N_\nu + d\left(\frac{x_0}{2} K_\nu + x_1 K_{\nu-1} + \cdots + x_{\nu-1} K_1\right). \tag{14}$$

These equations are solved for the q−2 (q−1) parameters, i.e., we obtain equations of the form

$$\sigma_1 = f_1(\sigma_q, \sigma_{q-1}), \ \sigma_2 = f_2(\sigma_q, \sigma_{q-1}), \ldots, \ \sigma_{q-2} = f_{q-2}(\sigma_q, \sigma_{q-1}). \tag{15}$$

The expressions obtained are substituted in the system's characteristic equation D(p) in order to eliminate part of the unknowns. As a result, we obtain an equation with two (or one) parameters σ_q, σ_{q-1}:

$$D(p, \sigma_q, \sigma_{q-1}) = a_n + a_{n-1}p + \ldots + a_0 p^n. \tag{16}$$

2. If we substitute $p = \chi e^{j\psi}$ in Eq. (16) and then so vary χ and ψ that the end of vector p in the p-plane slides along the boundary of the domain of given quantity (Fig. 1), we obtain a representation of this domain on the plane of parameters σ_q, σ_{q-1}.

3. Inside the plane of given quality (on the parameter plane) we choose a point, and from these values of σ_q and σ_{q-1} we determine the remaining parameters by formulas (15).

The determination of the parameters by the representation method guarantees system stability as well as the other requirements enumerated above on control quality.

The most advantageous form of the region of given quality is the six-sided one (Fig. 1) which gives the limiting value of the real roots of the characteristic equation and of the system's frequency and oscillation. Thanks to the existence of well-defined relationships between the parameters, introduced by Eqs. (14), the cutting off of region $\mu\eta g$ (Fig. 1), as a rule, is insignificant. Finally, the parameters of the given curve $x^0(t)$ must be matched with the parameters of the R_1, R_2, and α region.

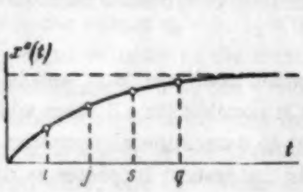

Fig. 2.

If high accuracy in reproducing the given process $x^0(t)$ is not required, one can simplify the computations by representing, not the domain of given quality, but the boundary of the region of stability (the imaginary axis of the p-plane). An even greater simplification occurs in the case when the system is stable for all practically possible values of the parameters to be determined. There is then no necessity of using the D-decomposition: all q parameters are found as a result of solving the system (14) of q equations, followed by a test for stability of the system.

It should be emphasized that the method allows one to determine a large number of parameters. In the Appendices, we give examples of synthesis with four and with three parameters to be varied.

3. On the Synthesis of Individual Links

By the synthesis of a link we understand the determination of its transfer function, its form (circuit) and its parameters. Without carrying out a detailed consideration here, we state that the method presented can also be used for the synthesis of individual links, including correcting links. The controller as a whole can be considered as an individual link; in this case, one determines the control law, the gain, etc. Links can be determined which are connected in complicated ways (not necessarily in series or in parallel) with the remaining portions of the system.

We first find the desirable transfer function and form of the link. Let a dynamic system be described by u equations with u generalized coordinates and definite disturbing stimuli. The last equation of the system, the equation of the link desired, has the form:

$$Q(p)\,x_l = R(p)\,x_g, \quad X(p) = \frac{x_g}{x_l} = \frac{Q(p)}{R(p)}, \tag{17}$$

where x_g and x_l are, respectively, the link's output and input coordinates and X (p) is its transfer function.

If we are given the desired process for one of the coordinates x_u, we solve, in operator form, the first u−1

equations for x_g and x_l and, after substituting in Eq. (17), we find the desired link's transfer function $X^0(p)$.

An exact reproduction of $X^0(p)$ is generally disadvantageous, due to its complexity; we therefore deal with a compromise solution which permits the quality condition to hold approximately for the simplified link circuit. If the degree of the numerator of $X^0(p)$ is greater than the degree of the denominator then, by separating the integral part, we obtain terms of the form A_0 and $A_\nu p_\nu$ which define the law of transformation of the input signal, and which are reproduced, with a known error, by differentiating links. If the representation of $X^0(p)$ is a proper fraction, then we first determine the characteristic in the time domain, $x^0(t)$. From this, using the condition of approximate coincidence of the time domain characteristics $x^0(t)$ and $x_{app}(t)$, we determine the order (form) of the transfer function of the simplified link $X_{app}(p) = x_{app}(t)$. To alleviate the choice of $X_{app}(p)$, one can set up a "dictionary" of standard links and their transfer functions.

The coefficients of the numerator and denominator of $X_{app}(p)$ are determined by using relationships (13) and (14) and the condition that one obtain the best point approximation of the actual process $x_u(t)$ to the given one when link $X_{app}(p)$ is connected in the circuit. For this, the condition of stability of the entire system must be met, where the system includes the link being determined. This latter requirement is met by constructing the region of stability (or of given quality) as was shown in section 2.

To simplify the calculations, Eqs. (14) can also be set up from the condition that $x_{app}(t)$ approximates to $x^0(t)$. In this latter case, the error of reproducing $x_u(t)$ will exceed the error in the approximation of $x_{app}(t)$ to $x^0(t)$. For the first error to lie within the limits of 10 to 12%, the second error, as a rule, must not exceed 3-4%; with this, of course, stability of the whole system must be guaranteed.

The circuit implementation of the simplified link with transfer function $X_{app}(p)$ is carried out either immediately, or by application of the ordinary methods of circuit theory.

Finally, we draw the following conclusions as to the basic advantages of the method of synthesis considered here.

1. In using the method, one obtains linear relationships between the representation coefficients a_k and b_k (in contradistinction to the method of integral estimates, for example) which makes it possible (in all cases when there is a linear relationship between the a_k and b_k and the parameters being sought) to determine any number of parameters. More than that, the accuracy of reproduction of a desirable process in the system increases as the number of parameters to be varied increases, with a relatively small increase in the amount of computational work required. Existing methods ordinarily allow one to determine no more than two parameters of the different links in a system.

2. For the parameters found by the synthesis method presented here, it is guaranteed that one obtain a stable system and the given quality indicators, defined both by the representation's denominator (domain of given quality) and by its numerator (point approximation to the desired transient response curve).

3. The parameters sought in the synthesis method considered here can appertain to different links of the system. One can find the form (circuit) and parameters of an individual link whose switching in determines the course of the process with given indications of quality. A link can be connected in a complex way (not necessarily in series or in parallel) with the rest of the system.

4. The computations in the method presented here are carried out in accordance with a series of rules (cf. section 2) which are based on the execution of simple mathematical operations: calculating the coefficients of polynomials, solving systems of linear algebraic equations and calculating the values of polynomials for certain values of a variable ω. Thanks to its lack of complexity, the method does not require special preparation for the computer. Since the course of the calculations is formulated as a series of algorithms, it can conveniently be turned over to a computing machine. The method can also be used for programing the synthesis problem for an electronic computer.

APPENDIX 1

We now determine the four gains in the regulation of the excitation of a synchronous generator by the derivatives of current and voltage. We determine the coefficients from a consideration of the transient response and the stability of small oscillations when the generator operates on a load via a long transmission line. The representation of the angle δ between the vectors of the generating station emf and the voltage of the receiving system is given by expression (1), where:

$$
\begin{aligned}
&n = 4,\ m = 2,\\
&b_0 = 0.287 - 0.389\,\beta_2 - 0.632\,\gamma_2,\\
&b_1 = 4.86 - 0.389\,\beta_1 - 0.632\,\gamma_1,\\
&b_2 = 1.11 - 0.389\,\beta_0 - 0.632\,\gamma_0,\\
&a_0 = (1.21 - 1.63\,\beta_2 - 2.65\,\gamma_2)\,10^{-2},\\
&a_1 = (20.5 - 1.63\,\beta_1 - 0.124\,\beta_2 + 2.65\,\gamma_1 + 0.201\,\gamma_2)\,10^{-2},\\
&a_2 = (18.7 - 1.63\,\beta_0 - 0.124\,\beta_1 + 29.3\,\beta_2 - 2.65\,\gamma_0 - 0.201\,\gamma_1 - 39.9\,\gamma_2)\,10^{-2},\\
&a_3 = 2.10 - 0.124 \cdot 10^{-2}\,\beta_0 + 0.293\,\beta_1 - 0.201 \cdot 10^{-2}\,\gamma_0 - 0.399\,\gamma_1,\\
&a_4 = 0.298 + 0.293\,\beta_0 - 0.399\,\gamma_0.
\end{aligned}
\tag{18}
$$

Here, γ_0 and β_0 are the gains when control is by deviations of the voltage and the current from their nominal values. By calculating them from the given static characteristics, we find that $\beta_0 = 1.0$ and $\gamma_0 = -15.0$.

To determine the gains when control is by the first and second derivatives of current and voltage, denoted, respectively, by β_1, β_2, γ_1 and γ_2, we start out from the following requirements: the system must be stable and have a degree of stability of $R_1 = 2$; the transient response must be monotonic (or with small overshoot) and practically without oscillations, the duration of the transient response is $T \approx 2.5$ sec, the maximum velocity of the transient response is $v \approx 2.5\ \text{sec}^{-1}$.

To satisfy these requirements, it is necessary that the following conditions hold.

1. The roots of the characteristic equation must lie inside the domain of given quality (Fig. 1) for which $R_1 = 2$, $\alpha = 120°$, $R_2 = 12$ and $H = 10$. The values of R_2 and H limit the real parts of the roots of the characteristic equation and the frequency of oscillation, and are arbitrarily taken here; α is determined by the oscillation of the system.

2. The curve of the transient response must be close to the exponential curve with Eq. (1.55) $(1-e^{-2t})$, giving the values $x_0 = 0$, $x_1 = 0.62$, $x_2 = 1.1$ and $x(\infty) = 1.55$ for $d = 0.25$ sec. This also guarantees that the maximum velocity of the transient response is $v \approx (x_1 - x_0)/d = 2.5\ \text{sec}^{-1}$. The duration of the transient response is determined both by the given curve and by the magnitude of R_1. The point $t_2 = 2d$ of the approximation to the desired curve is adopted from the condition that $t_2 = \tau_{max}$, where $\tau_{max} = 1/R_1$ is the maximum time constant component of the transient response. With this, a small overshoot is possible for $t > \tau_{max}$. To avoid such a possibility, one can increase the number of points of the approximation.

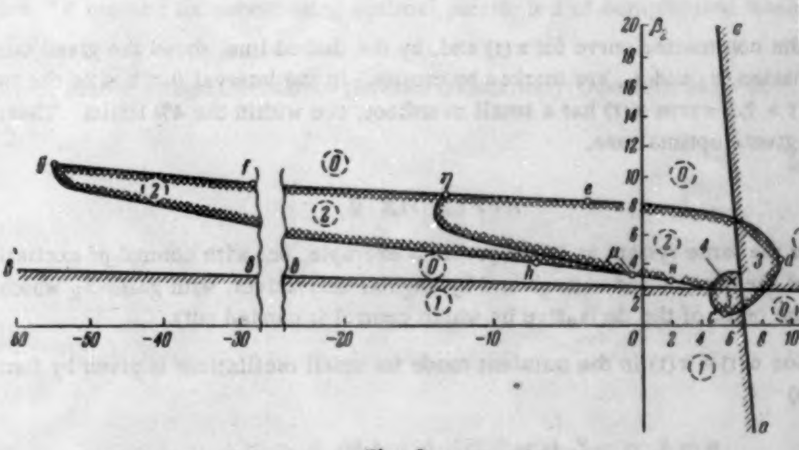

Fig. 3.

The calculations are carried out in the following order.

The expressions for N_1 and K_1 for $t = d$ and the expression for N_2 for $t = 2d$ are found from formulas (9) and (12):

$$
\begin{aligned}
N_1 &= (2.41 - 0.101\,\beta_1 - 1.22\,\beta_2 - 0.165\,\gamma_1 - 0.198\,\gamma_2)\,10^{-2},\\
K_1 &= (43.1 - 0.751\,\beta_1 + 7.19\,\beta_2 - 3.95\,\gamma_1 - 10.17\,\gamma_2)\,10^{-2},\\
N_2 &= (17.69 - 0.811\,\beta_1 - 4.86\,\beta_2 - 1.32\,\gamma_1 - 7.90\,\gamma_2)\,10^{-2}.
\end{aligned}
$$

For moments of time t_1 and t_2, one sets up system (14) of equations and, after substituting the expressions for N_1, N_2 and K_1 in them, one solves the system for γ_1 and γ_2:

$$\gamma_1 = -0.949\,\beta_1 - 2.67\,\beta_2 + 7.76, \quad \gamma_2 = -0.0435\,\beta_1 - 0.964\,\beta_2 + 1.26. \tag{19}$$

The expressions obtained for γ_1 and γ_2, as well as the values of γ_0 and β_0, are substituted in formulas (18) for the coefficients a_k, and these latter are then substituted in the characteristic polynomial given in (16).

After this, one maps the domain of given quality (a six-sided figure) in the p-plane on the plane of parameters β_1, β_2 by the method of D-decomposition, as shown on Fig. 3. Points a and b of the hexagon on the parameter plane correspond to lines a and b, while the remaining mapped and image points have the subscripts, respectively, of c,d, . . .,k.

In Fig. 8 is constructed also the image of a trapezoid in the plane p (Fig. 1, points f, g, h). As is evident, region $\eta g \mu$ is ignorable since in the plane β_1, β_2 it is not a working region.

Since the contender, in the domain of given quality in the β_1, β_2 plane, has marks on the four roots which are greater than the minimum marks (zero root) (Fig. 3) and since the characteristic equation is of fourth degree, the marked domain 4 (Fig. 3) is the region of given quality. Approximately in its center we choose a point (marked by the cross) and then obtain the values of the parameters $\beta_1 = 5.8$ and $\beta_2 = 2.8$. Then, by Eqs. (19), we find that $\gamma_1 = -5.23$ and $\gamma_2 = -1.69$.

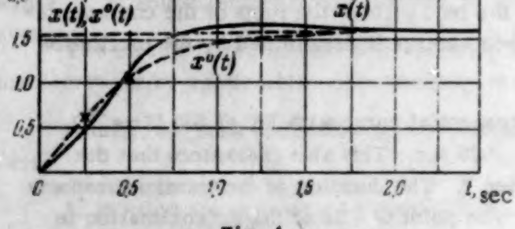

Fig. 4.

With this, the synthesis of the system, i.e., the determination of the gains, terminates.

To verify what we have done, we substitute the values found for β_1, β_2, γ_1 and γ_2 in the coefficients a_k of the characteristic equation and then compute the roots. We obtain two pairs of complex conjugate roots: $p_{1,2} = -2.01 \pm 1.18\,j$ and $p_{3,4} = -8.98 \pm 5.0\,j$.

These results show that the roots lie within the given region, i.e., the hexagon. From the roots which we now know we find the exact value of the time-domain function $x(t) = \delta(t)$, and we then construct the curve of the transient response.

Figure 4 shows the constructed curve for $x(t)$ and, by the dashed line, shows the given curve $x^0(t)$, on which the points of approximation, x_1 and x_2, are marked by crosses. In the interval $0 < t \leq 2d$ the two curves practically coincide. For $t > 2d$, curve $x(t)$ has a small overshoot, one within the 4% limits. Thus, the actual curve differs little from the given, optimal one.

APPENDIX 2

We now consider the same system as in the previous example, but with control of excitation by deviations from nominal values of the current and voltage and by angular derivatives, with gains σ_k which are to be determined ($k = 1, 2, 3$ is the order of the derivative by which control is carried out).

The representation $\delta(t) = x(t)$ in the transient mode for small oscillations is given by formula (1), where (for the nominal mode)

$$\begin{gathered} n = 4,\ m = 2,\ b_0 = 0.287,\ b_1 = 4.86,\ b_2 = 10.2, \\ a_0 = 1.2 \cdot 10^{-2},\ a_1 = 0.205 + 0.74\,\sigma_3,\ a_2 = 0.516 + 0.74\,\sigma_2,\ a_3 = 2.13 + 0.74\,\sigma_1,\ a_4 = 6.572. \end{gathered} \tag{20}$$

Since, in the plane of parameters σ_k, the system has a broad open region of stability, we determine the parameters sought from the condition that $\delta(t)$ approximate to the given process $\delta^0(t) = 1.55\,(1 - e^{-2t})$ at five points. In addition to the points $t_1 = 0$ and $t_5 = \infty$, in which both time-domain functions coincide for any values of the σ_k, we choose as the approximation points $t_2 = d$, $t_3 = 2d$, and $t_4 = 3d$, where $d = 0.25$ sec. Using formula (14), we set up for them the system of three equations whose solution gives the following values for the gains:

$$\sigma_1 = 7.13, \quad \sigma_2 = 1.3, \quad \sigma_3 = 6.5 \cdot 10^{-3}.$$

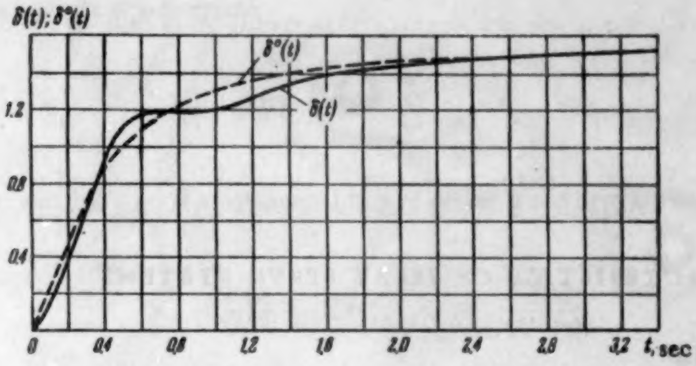

Fig. 5.

In view of the smallness of the product $\sigma_3\dddot{\delta}$, we take $\sigma_3 = 0$. [The derivative $\dddot{\delta}$ has order 7, which is clear from the expression for $\delta^0(t)$.] To verify the accuracy of reproduction of the given process we find, from formula (1), the time-domain function $\delta(t)$ with the values of σ_1, σ_2, and σ_3 obtained as the result of our computations. Figure 5 shows the given curve $\delta^0(t)$ and the actual curve $\delta(t)$. As is obvious, the maximum relative error is within 6%.

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FREQUENCY CHARACTERISTICS OF RELAY SERVO SYSTEMS

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A method is presented for constructing exact amplitude-frequency and phase-frequency characteristics of relay servo systems with arbitrary forms of external periodic stimuli. At the base of this method is the concept of the generalized characteristic of a relay system, a particular case of which was used in [4] for the investigation of periodic modes.

Examples are given which illustrate the method described.

In those cases when a relay servo system operates in the conditions of a periodic external stimulus, important quantities which characterize the quality of its operation are the maximum value of its output quantity and the phase shift between the output quantity and the external stimulus.

The dependence of these quantities on the frequency of the external periodic stimulus for a constant "amplitude" of the latter determines the frequency properties of the relay system.

The known approximate methods for constructing frequency characteristics, based on the harmonic balance method or its offshoots [1-3], are connected with a host of essential limitations.

Among these limitations are, for example, the assumption of harmonic form of the output quantity and external stimulus. Since these assumptions are not always fulfilled or justified, the approximate method can lead to unwarranted conclusions.

In the present paper, we introduce the concept of the frequency characteristic of a relay servo system, and present a method of constructing it which is not related to any limiting assumptions. This method uses the concept of the characteristic of a relay system [4]. By the frequency characteristic of a relay system we understand the dependence of the maximum value of the output quantity of the continuous part (or a segment of it) on the frequency of an external periodic stimulus of arbitrary form, and the dependence of the phase shift between the output quantity and the external stimulus on the frequency of the latter. With this, the maximum deviation of the external periodic excitation is assumed to be constant.

Preliminary Remarks

We consider a system consisting of a relay element and a continuous portion (Fig. 1a). The relay element's characteristic has the form shown in Fig. 2a, and the continuous portion is characterized by the transfer function

$$K(p) = \frac{P(p)}{Q(p)}, \tag{1}$$

or by the frequency characteristic

$$K(j\omega) = U(\omega) + jV(\omega) = K_0(\omega)\, e^{j\theta(\omega)}, \tag{2}$$

or, finally, by the time-domain characteristic

$$h(t) = c_{00} + \sum_{\nu=1}^{n} c_{\nu 0} e^{p_\nu t}. \tag{3}$$

In expressions (1) and (3), Q(p) is a polynomial in p of degree n and P(p) is either a polynomial or a transcendental function

$$c_{00} = \frac{P(0)}{Q(0)} = K(0), \quad c_{\nu 0} = \frac{P(p_\nu)}{p_\nu Q'(p_\nu)}, \tag{4}$$

and p_ν are the roots of the equation Q(p) = 0, assumed to be simple.

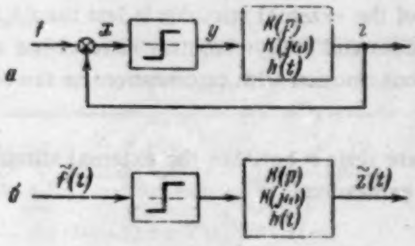


Fig. 1.

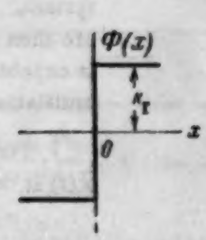


Fig. 2.

As in [4], we denote by $\tilde{z}(t)$ the steady-state quantity at the output of an open-loop relay system (Fig. 1b) when the input quantity varies periodically with frequency ω_0.* As was shown in [4], the expression for $\tilde{z}(t)$ can be given in various forms:

$$\tilde{z}(t) = k_r \left\{ c_{00} + \sum_{\nu=1}^{n} c_{\nu 0} \frac{2e^{p_\nu t}}{1 + e^{p_\nu \frac{\pi}{\omega_0}}} \right\}, \tag{5}$$

if the transfer function of the continuous part is given;

$$\tilde{z}(t) = \frac{4k_r}{\pi} \sum_{m=1}^{\infty} \frac{K_0[(2m-1)\omega_0]}{2m-1} \sin[(2m-1)\omega_0 t + \theta((2m-1)\omega_0)], \tag{6}$$

if the frequency characteristic of the continuous part is given;

$$\tilde{z}(t) = k_r \left\{ h(t) + \sum_{k=1}^{\infty} (-1)^k \Delta h\left[t + (k-1)\frac{\pi}{\omega_0}\right] \right\}, \tag{7}$$

if the time-domain characteristic of the continuous part is given.

The conditions for the existence of forced oscillations, engendered by the external stimulus

$$\tilde{f}(t) = A\tilde{f}_0(\omega_0 t - \varphi) \tag{8}$$

are defined by the relay system's characteristic (Fig. 3)

*We note that $\tilde{z}(t)$, for ω_0 = const, does not depend on the form of the input quantity.

$$J(\omega) = -\frac{\tilde{z}\left(\frac{\pi}{\omega}\right)}{\omega} - j\tilde{z}\left(\frac{\pi}{\omega}\right) = \operatorname{Re} J(\omega) + j \operatorname{Im} J(\omega) \tag{9}$$

by means of the simple constructions described in [4].

The critical value of the amplitude is

$$A_{cr} = |\operatorname{Im} J(\omega)|. \tag{10}$$

It depends both on the frequency of the external stimulus and on the form of J(ω).

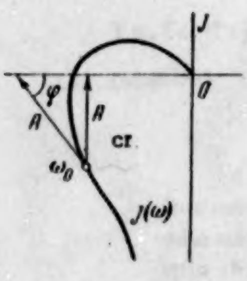


Fig. 3.

If the amplitude of the external stimulus is greater than A_{cr}, there then exists a mode of forced oscillation, with frequency of the external stimulus, in the relay system. If the amplitude of the external stimulus is less than A_{cr} ($A < A_{cr}$) there are then no forced oscillations and the system may have either a pulsed mode (if it is capable of autooscillation), motion with interruptions or the so-called subharmonic oscillations.

For $A > A_{cr}$, the phase shift φ between the external stimulus and the error $\tilde{x}(t)$ is determined by the expression

$$|\tilde{f}_0(\pi - \varphi)| = |\tilde{f}_0(-\varphi)| = \frac{A_{cr}}{A} \tag{11}$$

or by

$$\varphi = -\tilde{f}_0^{-1}\left(\frac{A_{cr}}{A}\right), \tag{12}$$

where $\tilde{f}_0^{-1}$ denotes the function inverse (reciprocal) to $\tilde{f}_0$.

Construction of the Frequency Characteristics

The frequency characteristic is comprised of the amplitude-frequency characteristic, defined as the dependence of the maximum of $\tilde{z}(t)$ on the frequency of the external periodic stimulus of arbitrary form (Fig. 4):

$$R(\omega) = \max|\tilde{z}(t)|, \tag{13}$$

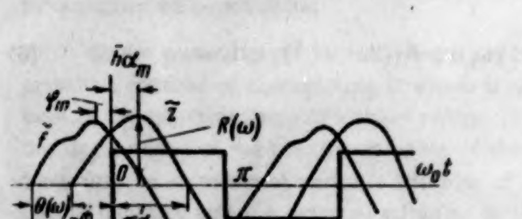


Fig. 4.

and the phase-frequency characteristic, defined as the phase shift between $\tilde{z}(t)$ and $\tilde{f}(t)$ (Fig. 4):

$$\theta_R(\omega) = \arg \tilde{f}(t) - \arg \tilde{z}(t) = -\varphi + \psi \tag{14}$$

for A = const. To determine the amplitude-frequency and phase-frequency characteristics, we introduce the concept of the generalized characteristic of a relay system

$$J(\alpha, \omega) = -\frac{\tilde{z}\left(\alpha \frac{\pi}{\omega}\right)}{\omega} - j\tilde{z}\left(\alpha \frac{\pi}{\omega}\right), \tag{15}$$

where α varies from 0 to 1. For $\alpha = 1$, the generalized characteristic coincides with relay system characteristic (9).

The real and imaginary parts of the generalized characteristic of a relay system

$$\operatorname{Re} J(\alpha,\ \omega) = -\frac{\dot{\tilde{z}}\left(\alpha\frac{\pi}{\omega}\right)}{\omega}, \qquad \operatorname{Im} J(\alpha,\ \omega) = -\tilde{z}\left(\alpha\frac{\pi}{\omega}\right) \tag{16}$$

may, on the basis of formulas (4), (6), and (7), also be presented in various forms, which are given in the table.

As α varies from 0 to 1, the imaginary part $\operatorname{Im}J(\alpha,\omega)$ defines the form of the periodic quantity $\tilde{z}(t)_{t=\alpha\frac{\pi}{\omega}}$, and the real part $\operatorname{Re}J(\alpha,\omega)$, in the scale of $1/\omega$, determines the form of its derivative. It can be shown that $J(\alpha,\omega)$ possesses the following properties:

$$\operatorname{Re} J(\alpha,\ \omega) = \frac{1}{\pi}\frac{\partial \operatorname{Im} J(\alpha,\ \omega)}{\partial \alpha}, \qquad J(0,\ \omega) = -J(1,\omega). \tag{17}$$

TABLE

$\operatorname{Re} J(\alpha,\omega)$	$\operatorname{Im} J(\alpha,\omega)$
$-\frac{4k_r}{\pi}\sum\limits_{m=1}^{\infty}\{\cos(2m-1)\alpha\pi\cdot U[(2m-1)\omega]- -\sin(2m-1)\alpha\pi\cdot V[(2m-1)\omega]\}$	$-\frac{4k_r}{\pi}\sum\limits_{m=1}^{\infty}\left\{\frac{\sin(2m-1)\alpha\pi}{2m-1}U[(2m-1)\omega]+ +\frac{\cos(2m-1)\alpha\pi}{2m-1}V[(2m-1)\omega]\right\}$
$-\frac{k_r}{\omega}\left\{\dot{h}\left(\alpha\frac{\pi}{\omega}\right)+ +\sum\limits_{k=1}^{\infty}(-1)^k\Delta\dot{h}\left(k-1+\alpha\right)\frac{\pi}{\omega}\right\}$	$-k_r\left\{h\left(\alpha\frac{\pi}{\omega}\right)+ +\sum\limits_{k=1}^{\infty}(-1)^k\Delta h\left(k-1+\alpha\right)\frac{\pi}{\omega}\right\}$
a) $-\frac{k_r}{\omega}\sum\limits_{\nu=1}^{n}c_{\nu 0}p_\nu\frac{2e^{\frac{\alpha\pi}{\omega}p_\nu}}{1+e^{\frac{\pi}{\omega}p_\nu}}$	$-k_r\left\{c_{00}+\sum\limits_{k=1}^{\infty}c_{\nu 0}\frac{2e^{\frac{\alpha\pi}{\omega}p_\nu}}{1+e^{\frac{\pi}{\omega}p_\nu}}\right\}$
for different and nonzero fields $p\nu$;	
б) $-\frac{k_r}{\omega}\left[c'_{00}+\sum\limits_{\nu=1}^{n-1}p_\nu c_{\nu 0}\frac{2e^{\frac{\alpha\pi}{\omega}p_\nu}}{1+e^{\frac{\pi}{\omega}p_\nu}}\right]$	$k_r\left\{\frac{(1-2\alpha)\pi}{2\omega}c'_{00}+\sum\limits_{\nu=1}^{n-1}c_{\nu 0}\left[1-\frac{2e^{\frac{\alpha\pi}{\omega}p_\nu}}{1+e^{\frac{\pi}{\omega}p_\nu}}\right]\right\}$
for the presence of single, zero field $P_n = 0$.	

Note to table. In the last formulas, $c'_{00} = P(0)/Q'(0)$.

To determine the amplitude-frequency and the phase-frequency characteristics, we use the relay system's characteristic $J(\omega) = J(1,\omega)$. From each point $\omega = \omega_0$ of this characteristic we construct, from the expression for the generalized characteristic $J(\alpha,\omega)$, segments of curves by varying α until these curve segments intersect the axis of ordinates for $\alpha = \alpha_m$ and the axis of abscissas for $\alpha = \alpha_0$ (on Fig. 5, the values of α_m and α_0 are denoted by α_{im} and α_{i0}, respectively). For $\alpha = \alpha_m$

$$\operatorname{Re} J(\alpha_m,\ \omega) = -\frac{\dot{\tilde{z}}\left(\alpha_m\frac{\pi}{\omega}\right)}{\omega} = 0,$$

$$|\operatorname{Im} J(\alpha_m,\ \omega)| = \left|\tilde{z}\left(\alpha_m\frac{\pi}{\omega}\right)\right| = \max\left|\tilde{z}\left(\alpha\frac{\pi}{\omega}\right)\right|; \tag{18}$$

for $\alpha = \alpha_0$,

$$\operatorname{Im} J(\alpha_0, \omega) = -\tilde{z}\left(\alpha_0 \frac{\pi}{\omega}\right) = 0,$$

$$|\operatorname{Re} J(\alpha_0, \omega)| = \frac{\left|\tilde{z}\left(\alpha_0 \frac{\pi}{\omega}\right)\right|}{\omega} = \max \frac{\left|\tilde{z}\left(\alpha \frac{\pi}{\omega}\right)\right|}{\omega}. \tag{19}$$

In accordance with (14) and (18), the amplitude-frequency characteristic $R(\omega)$ will equal

$$R(\omega) = |\operatorname{Im} J(\alpha_m, \varphi)|. \tag{20}$$

Thus, the segments of the axis of ordinates cut off by the curves $J(\alpha, \omega)$ for $\omega = \text{const}$ and α varied equal to the ordinates of the amplitude-frequency characteristic for $\omega = \text{const}$. Obviously, $R(\omega)$ exists only for those frequencies for which $A \geq A_{cr}$ [4]. We note that $R(\omega)$ does not depend on the form of the external stimulus.

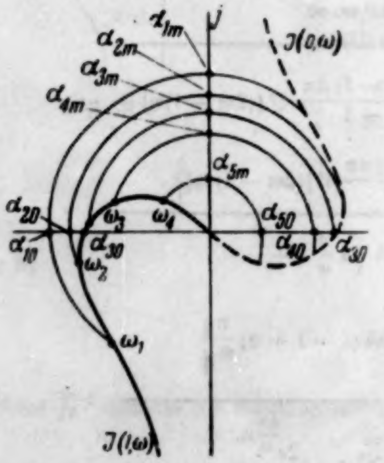


Fig. 5.

The phase-frequency characteristic $\theta_R(\omega)$ can be determined as the algebraic sum of the phase shifts of $\tilde{z}(t)$ and $\tilde{f}(t)$ with respect to the error $x(t)$ (Fig. 4), i.e.,

$$\theta_R(\omega) = \psi - \varphi = \pi(\alpha_0 - 1) - \varphi$$

or, according to (12),

$$\theta_R(\omega) = \pi(1 - \alpha_0) + \tilde{f}^{-1}\left(\frac{A_{cr}}{A}\right). \tag{21}$$

Thus, the phase-frequency characteristic is determined by the values of $\alpha = \alpha_0$ for which $J(\alpha, \omega)$ intersects the axis of abscissas; it depends on the amplitude and the form of the external periodic stimulus.

If the forms of $\tilde{f}(t)$ and $\tilde{z}(t)$ are almost symmetric about their maximum values, the phase shift may then be determined approximately from the distance between their maxima [5] (Fig. 4). In this case, by setting $\varphi_m = \varphi + \pi/2$, we can write

$$\theta_R(\omega) \approx \theta_m = \pi\alpha_m - \varphi_m = \pi\alpha_m - \varphi - \frac{\pi}{2} = \pi\left(\alpha_m - \frac{1}{2}\right) + \tag{22}$$

$$+ \tilde{f}^{-1}\left(\frac{A_{cr}}{A}\right).$$

Formulas (20) and (22) are basic for the graphicoanalytic construction of the amplitude-frequency and phase-frequency characteristics, i.e., the frequency characteristics of relay systems. We note that, for the simplest relay systems, the frequency characteristics can be given in analytic form.

<u>Example.</u> We consider a relay servo system whose continuous part has a transfer function of the form:

$$K(p) = \frac{k_1}{p(T_1 p + 1)(T_2 p + 1)}.$$

From the table we determine $J(\alpha, \omega)$. In this case, the poles of $K(p)$ are known, so it is convenient to express $J(\alpha, \omega)$ in terms of the transfer function in the form

$$\operatorname{Re} J(\alpha, \omega) = -\frac{2k_r k_1}{\omega(T_1 - T_2)}\left(\frac{T_1 - T_2}{2} - T_1 \frac{e^{-\alpha\frac{\pi}{\omega T_1}}}{1 + e^{-\frac{\pi}{\omega T_1}}} + T_2 \frac{e^{-\alpha\frac{\pi}{\omega T_2}}}{1 + e^{-\frac{\pi}{\omega T_2}}}\right) \tag{23}$$

$$\operatorname{Im} J(\alpha, \omega) = k_r k_1 \left\{ \frac{1-2\alpha}{2\omega}\pi + (T_1 + T_2) - \frac{2}{T_1 - T_2}\left(T_1^2 \frac{e^{-\alpha\frac{\pi}{\omega T_1}}}{1 + e^{-\frac{\pi}{\omega T_1}}} - \right.\right.$$

$$\left.\left. - T_2^2 \frac{e^{-\alpha\frac{\pi}{\omega T_2}}}{1 + e^{-\frac{\pi}{\omega T_2}}} \right)\right\}. \tag{24}$$

We assume that the relay system has the following parameters: $k_r k_1 = 75$, $T_1 = 0.05$, and $T_2 = 0.03$. With these values, $J(\omega) = J(1,\omega)$ will have the form shown on Fig. 6 [here the representation $J(0,\omega) = -J(\omega)$ is used].

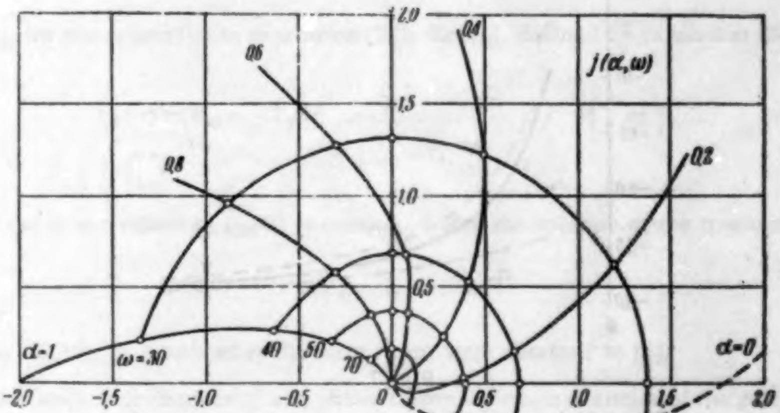


Fig. 6.

By setting ω = const and letting α vary from 0 to 1, we determine the curves $J(\alpha,\omega)$ which join points with equal ω on the characteristics $J(1,\omega)$ and $J(0,\omega)$. The values of α for which the $J(\alpha,\omega)$ intersect the coordinate axes define α_m and α_0, and the values of the ordinates equal to $R(\omega)$.

By being given, for example, the value $A = 0.2$, and by using the dependence of A_{cr} on ω (Fig. 7), we find that

$$\varphi = -\tilde{J}^{-1}\left(\frac{A}{A_{cr}}\right),$$

and then, from formula (20), we find $\theta_R(\omega)$. The functions $R_0(\omega)$ and $\theta(\omega)$ are given on Fig. 7 for triangular and rectangular forms of external stimuli. The frequency characteristics exist for those values of ω for which $A \geq A_{cr}$.

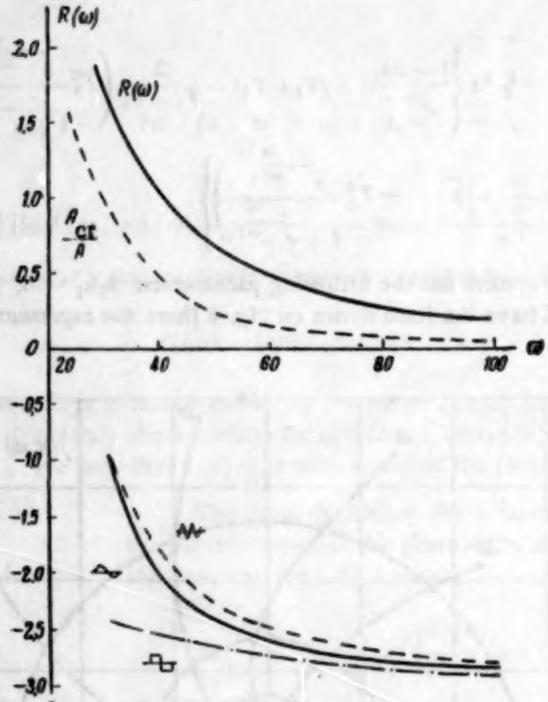
We now assume that $T_2 = 0$. The transfer function of the relay system will then have the simpler form:

$$K(p) = \frac{k_1}{p(T_1 p + 1)}. \tag{25}$$

In this case, $R(\omega)$ and $\theta_R(\omega)$ can be expressed analytically. Indeed, we find from (23) and (24), for $T_2 = 0$, that

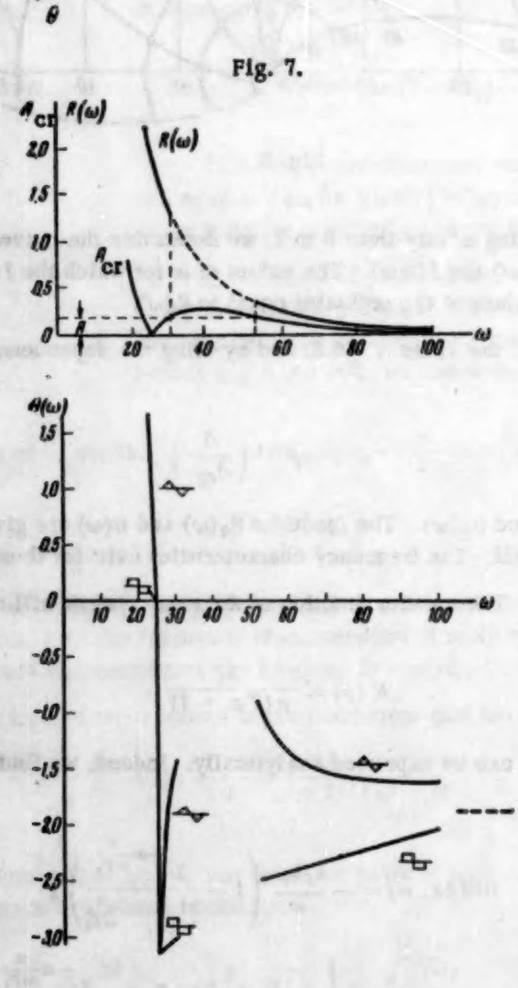
$$\operatorname{Re} J(\alpha, \omega) = -\frac{k_r k_1}{\omega}\left(1 - \frac{2e^{-\alpha\frac{\pi}{\omega T_1}}}{1 + e^{-\frac{\pi}{\omega T_1}}}\right), \tag{26}$$

$$\operatorname{Im} J(\alpha, \omega) = k_0 k_1 T_1 \left\{1 - \left(\alpha - \frac{1}{2}\right)\frac{\pi}{\omega T_1} - \frac{2e^{-\alpha\frac{\pi}{\omega T_1}}}{1 + e^{-\frac{\pi}{\omega T_1}}}\right\}. \tag{27}$$



R(ω)
2.0
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
θ
A_cr/A
20
40
60
80
100
ω

Fig. 7.



A_cr R(ω)
R(ω)
A_cr
A
2.0
1.5
1.0
0.5
0
20
40
60
80
100
ω
θ(ω)
1.5
1.0
0.5
0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
10
20
30
40
50
60
80
100
ω

Fig. 8.

By letting $\mathrm{Re}J(\alpha,\omega)$ vanish, we find $\alpha = \alpha_m$:

$$\alpha_m = \frac{\omega T_1}{\pi} \ln \frac{2}{1 + e^{-\frac{\pi}{\omega T_1}}} \tag{28}$$

By substituting this value in $\mathrm{Im}J(\alpha,\omega)$, we get

$$R(\omega) = k_r k_1 T_1 \left(-\frac{\pi}{2\omega T_1} - \ln \frac{2}{1 + e^{-\frac{\pi}{\omega T}}} \right). \tag{29}$$

To determine θ_R we use approximate expression (22); for α_m, defined by expression (28), we will have

$$\theta_R(\omega) \approx \theta_m - \omega T_1 \ln \frac{2}{1 + e^{-\frac{\pi}{\omega T_1}}} - \frac{\pi}{2} + \mathcal{F}^{-1}\left(\frac{A_{cr}}{A}\right). \tag{30}$$

Computation of the exact values of $\theta_R(\omega)$ is connected with the solution of the transcendental equation

$$\mathrm{Im}\, J(\alpha_0, \omega) = 0 \tag{31}$$

in α_0. Equations (29) and (30), in a somewhat different form, were obtained in [5].

Figure 8 gives the amplitude-frequency and phase-frequency characteristics of the relay system under consideration for sinusoidal, triangular and rectangular forms of external stimulus $\mathcal{F}(t)$, constructed from formulas (29) and (30) for $k_r k_1 = 75$ and $T_1 = 0.05$.

The author wishes to thank N.A. Korolev for discussing the results and for aid with the computations.

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ANALYSIS OF TRACKING FAILURE

IN AUTOMATIC CONTROL SYSTEMS

IN THE PRESENCE OF FLUCTUATING NOISE

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The failure of tracking in automatic control systems under the action of intense fluctuating noise is investigated by means of the Fokker-Planck equation. The Ritz-Galerkin method is used to solve the boundary problem where, by a failure, we understand an increase of the error in the system above some definite quantity. For a system with a smoothing circuit in the form of an integrator, formulas are found for the probabilities of absence of failures in the system as functions of the system parameters, noise level and time. The results are used in the analysis of the noise stability of the AFC (automatic frequency control) system of a receiver of continuous signals.

In the investigation of the processes in automatic control systems, one frequently bypasses the fact, due to the mathematical difficulties involved, that the element for isolating the error signal (the discriminator) has a narrow range of linearity in the majority of systems. This is true, in particular, of the servo systems of radio engineering.

Thus, in heterodyne automatic frequency control (AFC) systems [1], the parameter in the input signal which is tracked (followed) is its average frequency. Due to the narrowness of the signal spectrum as compared to the average frequency, the linear portion of the frequency discriminator's characteristic is also narrow, its width being approximately equal to the width of this spectrum. In automatic range-finding devices [2, 3], one follows the time position of a pulse whose duration is significantly less than the repetition period. With this, the linear portion of the time discriminator's characteristic is roughly equal to the pulse duration. Similar cases occur in AFC systems of television scanners, in various servo systems with phase comparisons of signals, etc.

The number of such examples can be greatly multiplied. In all these cases, the nonlinearity is unavoidable in principle and, for an error exceeding a certain limit, the feedback path is opened and tracking failure ensues. One of the causes of failure may be the presence of intensive fluctuating noise (for example, internal noise of the receiver) which, strictly speaking, cannot be assimilated to an equivalent variation of the parameter being tracked [3].

The aim of this paper is to analyze random processes in servo systems with nonlinear discriminators in the presence of intense fluctuating noise. It turns out to be convenient, in view of the nonlinearity of the problem, to use the Fokker-Planck equation for the probability density of the system errors in order to find the most interesting characteristics.

1. Mathematical Posing of the Problem

The block schematic of the servo systems to be analyzed is given in Fig. 1, and is roughly divided into two parts, namely, a discriminator and, in the other part, an amplifying-smoothing circuit together with the driven devices. Here, $u_1(t)$ and $u_3(t)$ are the voltages applied to the discriminator and $u_2(t)$ is its output voltage; $x(t)$

and z(t) are the input and output values, respectively, of the parameter being tracked and y = x−z is the current value of the error. Thus, in an AFC system we understand by x, y, and z the values of the corresponding frequencies and, in a radar range finder, the time intervals. The dual system of notation is introduced in order to emphasize the fact that, between the voltages $u_1(t)$, $u_2(t)$ and their parameters x and y, there is not a single-valued correspondence. Thus, $u_2(t)$ is proportional to y only in the absence of noise and in a narrow range of values of y.

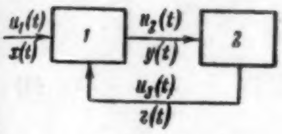

Fig. 1.

We now consider the case, of practical interest, when part 2 of the scheme in Fig. 1, which ordinarily contains some fundamental inertia, is an integrator with transfer function K_v/p. The generalization of the following analysis to the case of a system with a higher order of astatism is possible, but leads to a significant increase in computational difficulty. The servo system's integrator is described by the differential equation

$$\frac{dz}{dt} = K_v u_2(t). \tag{1}$$

Here, $u_2(t)$ is obtained from $u_1(t)$ and $u_3(t)$ by means of various linear and nonlinear operations intended to determine the sign and the magnitude of y. Thus, in an AFC system, these operations might include the parallel amplification of the signal of two tuned circuits, amplitude detection and subtraction of one result from the other; in a range-finder, they might include passage of the signal through two offset gates, integration within the limits of each of these and, again, subtraction of the results, etc.

We shall assume that the processes in discriminator 1 (Fig. 1) occur significantly more rapidly than those in the closed circuit as a whole. Then, to investigate the properties of $u_2(t)$, one need not consider the transient response in the discriminator induced by variations in y(t), but may characterize $u_2(t)$ only by the mean value $\overline{u_2(t)} = a\{y\}$ and the spectral density in the low-frequency range

$$S\{y\} = \int_{-\infty}^{+\infty} \left[\overline{u_2(t)\, u_2(t+\tau)} - \overline{u_2(t)}^2\right] d\tau,$$

which are functions of the current value of y; a{y} is called the discrimination characteristic; a typical form of it is shown in Fig. 2. A failure of tracking is observed if the error moves off the "slope" of this characteristic (for example, beyond the point −c/2, +c/2). S{y} characterizes the noise component in $u_2(t)$; in view of the assumptions made above, this may be considered as white noise where spectral density varies jointly with y. A typical form of S{y} for an AFC system is given in Fig. 3. Finding these two quantities for a discriminator of each concrete form is frequently a very difficult task [2]. If we consider these functions as being known, we may write $u_2(t)$ in the form

$$u_2(t) = a\{y\} + \sqrt{S\{y\}}\,\xi(t), \tag{2}$$

where ξ(t) is white noise with unit spectral density. By substituting (2) in (1) we get

$$\frac{d}{dt}\, y(t) = \left[\frac{dx}{dt} - K_v a\{y\}\right] + K_v \sqrt{S\{y\}}\,\xi(t). \tag{3}$$

It is known from the theory of random processes [4, 5] that, if the function sought y(t) is subject to a stochastic equation of the form*

$$\frac{dy}{dt} = A(t,\, y(t)) + B(t,\, y(t))\,\xi(t), \tag{4}$$

*In the mathematical literature on such cases, one frequently uses the concept of a Wiener process, i.e., an integral of white noise.

then the probability density W(t,y) of this function satisfies the differential equation (the Fokker-Planck equation)

$$\frac{\partial W(t, y)}{\partial t} = -\frac{\partial}{\partial y}\{A(t, y) W(t, y)\} + \frac{1}{2}\frac{\partial^2}{\partial y^2}\{B^2(t, y) W(t, y)\}. \tag{5}$$

By comparing (3) with (4) and (5), and by denoting

$$\frac{dx}{dt} = V, \ K_v a\{y\} = A(y), \ K_v\sqrt{S\{y\}} = B(y), \tag{6}$$

we have the following equation for the probability density of the error

$$\frac{\partial W(t, y)}{\partial t} = \frac{\partial}{\partial y}\{(A(y) - V) W(t, y)\} + \frac{1}{2}\frac{\partial^2}{\partial y^2}\{B^2(y) W(t, y)\}. \tag{7}$$

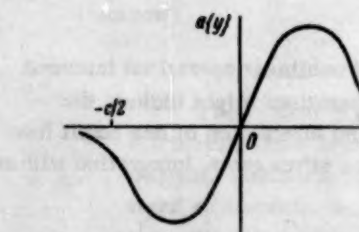

Fig. 2.

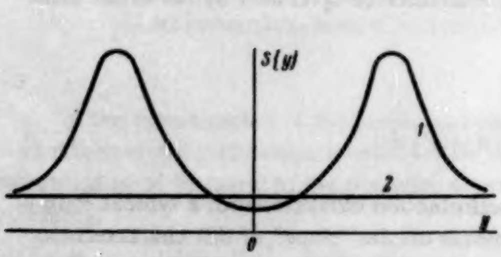

Fig. 3. Output spectral density of an AFC system as a function of the error: 1) low level of fluctuations; 2) high level of fluctuations.

The mathematical and physical literature on diffusion equations is very extensive (cf., for example, [6, 4, 7]). In the theory of oscillations, work on this question goes back to [8]. This apparatus was applied to problems of servo systems in [9], where the questions as to failures were first considered rigorously.

As follows from physical considerations, and also from a more rigorous analysis [10], the noise component of $u_2(t)$ can be considered as white noise only when the effective time constant of the dynamic system is significantly greater than the correlation interval of the noise voltage. When the noise level is low, this condition is sometimes poorly met in servo systems, but as the noise increases, the transmission factor of the discriminator $K_d = \frac{d}{dy} a\{y\}|_{y=0}$ generally falls, and the effective time constant is increased. In the study of the questions of tracking failures, the case of high noise levels is the more interesting one. Thus, the conditions for applicability of the diffusion equations hold precisely when, in view of the nonlinearity of the problem, the use of this apparatus is the most advantageous.

In the linear theory, the stationary random process at the servo system's output is ordinarily characterized by its mean square error. In the case of a nonlinear discrimination characteristic, of the form shown in Fig. 2, this process always remains nonstationary. This is explained by the probability of tracking failure which, by turning the system into a nonclosed integrator, leads to arbitrary wanderings of the value of y(t) with ever-increasing dispersion. To describe such a process, one must introduce other characteristics. Frequently interesting are the probability of failure from the moment of switching in of the noise up to some succeeding moment, and also the average (over the ensemble) of the time until failure. By the failure of the system at time $\underline{t}$ we shall understand that the magnitude of y(t) has exceeded some fixed level, $y_1 = -c/2$ or $y_2 = +c/2$, with the condition that the magnitude was found in the interval $[-c/2, +c/2]$ and had never left this interval until time $\underline{t}$. This leads [5] to the boundary problem for the function W(t,y) in which, at the boundaries, are given the boundary conditions

$$W(t, -\tfrac{1}{2}c) = 0, \quad W(t, +\tfrac{1}{2}c) = 0$$

(the "absorbing boundary" conditions). We choose the boundaries at the boundaries of the selected domain (passband of the basic amplifier in an AFC system or the gate cutting off the receiver in a range-finder) right at the "slopes" of the discrimination characteristic. In contradistinction to [9], this characteristic may be of any form. The probability P(t) of no failure up to time $\underline{t}$ is expressed in terms of the solution of the boundary problem as

$$P(t) = \int_{-1/2c}^{+1/2c} W(t,\ y)\,dy, \tag{8}$$

and the distribution law for the times of first failure has the form:

$$p(t) = -\frac{dP(t)}{dt} = -\frac{d}{dt}\int_{-1/2c}^{+1/2c} W(t,\ y)\,dy. \tag{9}$$

With this, the mean time until failure equals

$$T = \int_0^\infty tp(t)\,dt = \int_0^\infty P(t)\,dt = \int_0^\infty \int_{-1/2c}^{+1/2c} W(t,\ y)\,dydt. \tag{10}$$

The functions W(t,y), P(t), p(t) and T quite sufficiently characterize the process of servo system failure under the action of noise.

2. Setting Up the Equations for the Eigenvalues

Let the quantity to be tracked be constant (x = const) and the diffusion coefficient B be independent of y ($B(y) = B_0$). It can be shown that, with increasing noise, the dependence of B on y vanishes (Fig. 3). In section 4 we shall discuss these assumptions. We consider the error at the initial moment to be zero. It is then required to solve the equation

$$\frac{\partial W(t,\ y)}{dt} = \frac{B_0^2}{2}\frac{\partial^2 W(t,\ y)}{\partial y^2} + \frac{\partial}{\partial y}\{A(y)\,W(t,\ y)\} \tag{11}$$

with the auxiliary conditions

$$W\left(t,\ -\tfrac{1}{2}c\right) = 0,\ \ W\left(t,\ +\tfrac{1}{2}c\right) = 0,\ \ W(0,\ y) = \delta(y),$$

where δ(y) is the Dirac delta-function. We seek a solution in the form

$$W(t,\ y) = \sum_{i=1}^{\infty} \overline{C}_i e^{-\lambda_i t}\,Y_i(y), \tag{12}$$

where the $\overline{C}_i$ are defined by the initial conditions. We have arrived at a problem in eigenfunctions (e.f.) and eigenvalues (e.v.), described by the equation

$$\frac{B_0^2}{2}\,Y''(y) + \{A(y)\,Y(y)\}' = -\lambda Y(y). \tag{13}$$

Since there is no exact method of solving Eq. (13) for A(y) of arbitrary form, we choose, in accordance with the Ritz-Galerkin [11] approximation method, a system of coordinate functions which form a complete set on the given segment and which satisfy the boundary conditions. In such a system, we can conveniently choose the functions

$$\varphi_1 = \cos\frac{\pi}{c}\,y,\quad \varphi_2 = \sin\frac{2\pi}{c}\,y,\ \ldots,\ \varphi_{2n-1} = \cos\frac{(2n-1)\,\pi}{c}\,y,$$
$$\varphi_{2n} = \sin\frac{2n\pi}{c}\,y,\ \ldots, \tag{14}$$

which are the ef for the equation

$$\frac{B_0^2}{2} Y''(y) = -\lambda Y(y), \tag{15}$$

to which (13) degenerates with increasing noise, i.e., when $A(y) \to 0$. If we discard the odd φ_k from considerations of symmetry, we get $Y(y)$ in the form

$$Y(y) = \sum_{k=1}^{n} C_{2k-1} \cos \frac{(2k-1)\pi}{c} y. \tag{16}$$

By expanding $A(y)$ in a Fourier series in the complete system (for odd functions) $\varphi_2, \varphi_4, \ldots, \varphi_{2k}, \ldots$, by then substituting $A(y)$ and $Y(y)$ in series form in (13) and then by equating coefficients of φ_{2k-1} ($k = 1,2, \ldots, n$), we obtain a system of equations in the C_{2k-1} ($k = 1,2, \ldots, n$):

$$\begin{gathered}
\left[\left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{\pi}{c} \frac{A_1}{2} - \lambda\right] C_1 + \frac{\pi}{c} \frac{A_1 - A_2}{2} C_3 + \cdots + \frac{\pi}{c} \frac{A_{n-1} - A_n}{2} C_{2n-1} = 0, \\
-\frac{3\pi}{c} \frac{A_1 + A_2}{2} C_1 + \left[\left(\frac{3\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{3\pi}{c} \frac{A_3}{2} - \lambda\right] C_3 + \cdots \\
\cdots + \frac{3\pi}{c} \frac{A_{n-2} - A_{n+1}}{2} C_{2n-1} = 0, \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
-\frac{(2n-1)\pi}{c} \frac{A_{n-1} + A_n}{2} C_1 - \frac{(2n-1)\pi}{c} \frac{A_{n-2} + A_{n+1}}{2} C_3 + \cdots \\
\cdots + \left[(2n-1)^2 \left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{(2n-1)\pi}{c} \frac{A_{2n-1}}{2} - \lambda\right] C_{2n-1} = 0,
\end{gathered} \tag{17}$$

where

$$A_l = \frac{2}{c} \int_{-1/2 c}^{+1/2 c} A(y) \sin \frac{2l\pi}{c} y\, dy. \tag{18}$$

For (17) to have a nontrivial solution, it is necessary that its determinant equal zero:

$$\begin{vmatrix}
\left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{\pi}{c} \frac{A_1}{2} - \lambda & \frac{\pi}{c} \frac{A_1 - A_2}{2} & \cdots & \frac{\pi}{c} \frac{A_{n-1} - A_n}{2} \\
-\frac{3\pi}{c} \frac{A_1 + A_2}{2} & \left(\frac{3\pi}{c}\right)^2 \frac{B_0^2}{c} - \frac{3\pi}{c} \frac{A_3}{2} - \lambda & \cdots & \frac{3\pi}{c} \frac{A_{n-2} - A_{n+1}}{2} \\
\cdots & \cdots & \cdots & \cdots \\
-\frac{(2n-1)\pi}{c} \frac{A_{n-1} + A_n}{2} & -\frac{(2n-1)\pi}{c} \frac{A_{n-2} + A_{n+1}}{2} & \cdots & (2n-1)^2 \left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{(2n-1)\pi}{c} \frac{A_{2n-1}}{2} - \lambda
\end{vmatrix} = 0. \tag{19}$$

We have thus obtained an equation for the n first e.v. We now choose the number of coordinate functions. Since the fundamental result of the analysis must be the probability of no failures up to time t, expressed by (8), we estimate the accuracy with which a finite number of coordinate functions can satisfy the initial conditions in the sense of this relationship. The function $W(0,y)$ is expanded in a series of terms of the chosen coordinate functions in the form

$$W(0, y) = \delta(y) = \frac{2}{c} \sum_{k=1}^{\infty} \cos \frac{(2k-1)\pi}{c} y. \tag{20}$$

By integrating this equation over the entire interval $[-c/2, +c/2]$, we get

$$P(0) = 1 = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k-1}.$$

By choosing two coordinate functions we replace the series in the right member by the quantity $(4-\pi)[1-(1/3)] \approx 0.85$, i.e., we get a 15% deficiency, while by choosing three functions we obtain a 10% surplus. From considerations of continuity, it is clear that the approximate values $P^{(2)}(t)$ and $P^{(3)}(t)$, obtained with two and three coordinate functions, respectively, include the exact expression between themselves at the initial moment of time. Therefore, by carrying out the computations in parallel for n = 2 and n = 3, and by choosing as P(t) the arithmetic average of $P^{(2)}(t)$ and $P^{(3)}(t)$, we obtain a 2% accuracy in satisfying the initial conditions and a satisfactory accuracy in the approximation of P(t) for any t. A small number of coordinate functions does not make it possible to determine W(t,y), even for $t \to 0$. However, the desired probability density is easily found by solving the linear problem $(A(y) = K_v K_d y)$ on the infinite line since, in the first moments, the probability of the error going beyond the linear portion of the discriminator, not to mention reaching the boundary, is very small. It is easily shown that

$$\lim_{t\to 0} W(t, y) = \frac{1}{\sqrt{\frac{\pi B_0^2}{K_v K_d}\left(1-e^{-2K_v K_d t}\right)}} \exp\left\{-\frac{y^2}{\sqrt{\frac{B_0^2}{K_v K_d}\left(1-e^{-2K_v K_d t}\right)}}\right\}. \tag{21}$$

3. Finding the Eigenvalues and Setting Up the Solution

By taking (19) for n = 2 and by seeking the solution in the form of a series in powers of $1/B_0^2$, we obtain approximations for the first two e.v.

$$\lambda_1^{(2)} = \left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \left(\frac{\pi}{c}\right)\frac{A_1}{2} + \frac{3(A_1^2 - A_2^2)}{16B_0^2},$$

$$\lambda_3^{(2)} = \left(\frac{3\pi}{c}\right)^2 \frac{B_0^2}{2} - \left(\frac{3\pi}{c}\right)\frac{A_3}{2} - \frac{3(A_1^2 - A_2^2)}{16\,B_0^2} \tag{22}$$

(the superscript indicates the number of coordinate functions used).

For n = 3 we get

$$\lambda_1^{(3)} = \left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} + \left(\frac{\pi}{c}\right)\frac{A_1}{2} + \frac{3(A_1^2 - A_2^2)}{16B_0^2} + \frac{5(A_2^2 - A_3^2)}{48B_0^2},$$

$$\lambda_3^{(3)} = \left(\frac{3\pi}{c}\right)^2 \frac{B_0^2}{2} - \left(\frac{3\pi}{c}\right)\frac{A_3}{2} - \frac{3(A_1^2 - A_2^2)}{16B_0^2} + \frac{15(A_1^2 - A_4^2)}{32B_0^2}, \tag{23}$$

$$\lambda_5^{(3)} = \left(\frac{5\pi}{c}\right)^2 \frac{B_0^2}{2} - \left(\frac{5\pi}{c}\right)\frac{A_5}{2} - \frac{15(A_1^2 - A_4^2)}{32B_0^2} - \frac{5(A_2^2 - A_3^2)}{48B_0^2}.$$

We remark that the terms for the partial series (22) and (23) which include the A_l have one and the same order for different λ_{2k-1}, and that the first terms increase as $(2k-1)^2$. Consequently, in comparison with the degenerate case of (15), only the first few e.v. are subject to "perturbation," while the higher ones remain virtually invariant. This is explained by the fact that the higher e.v., which form a sharp peak in probability density function (21) in the first moments of time, are damped simultaneously with the dissolving of this peak until the moment when the inertia of the feedback, expressed by the coefficients A_l, comes into play. Therefore, the variation of the feedback parameters has very little effect on the speed of damping of the higher e.v.

Having the e.v., we can set up the solution. Let $C_{2i-1,2j-1}^{(n)}$ be the coefficient of the (2i−1)-st coordinate

function for the (2j−1)-st e.v. (n = 2,3, i,j = 1, . . .,n). Then, for n = 2, we obtain from (16) and (22), with initial conditions (20) taken into account,

$$C_{11}^{(2)} = \frac{2}{c} - \frac{A_1 - A_2}{4\pi B_0^2}, \quad C_{31}^{(2)} = \frac{3(A_1 + A_2)}{4\pi B_0^2},$$

$$C_{13}^{(2)} = \frac{A_1 - A_2}{4\pi B_0^2}, \quad C_{33}^{(2)} = \frac{2}{c} - \frac{3(A_1 + A_2)}{4\pi B_0^2}. \tag{24}$$

Here we have discarded terms of the order of $1/B_0^4$. We obtain the coefficients $C_{2i-1,2j-1}^{(3)}$ analogously. Finally, taking (12), (24) and the expressions for the $C_{2i-1,2j-1}^{(3)}$ into account, we obtain, for W(t,y),

$$W(t, y) = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{C_{2i-1,2j-1}^{(2)} + C_{2i-1,2j-1}^{(3)}}{2} e^{-\lambda_{2j-1} t} \cos \frac{(2i-1)\pi}{c} y =$$

$$= \left[\left(\frac{2}{c} - \frac{A_1 - A_2}{4\pi B_0^2} - \frac{A_2 - A_3}{24\pi B_0^2} \right) \cos \frac{\pi}{c} y + \frac{3(A_1 + A_2)}{4\pi B_0^2} \cos \frac{3\pi}{c} y + \right.$$

$$\left. + \frac{5(A_2 + A_3)}{24\pi B_0^2} \cos \frac{5\pi}{c} y \right] \exp\left\{ - \left[\left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{\pi}{c} \frac{A_1}{2} \right] t \right\} +$$

$$+ \left[\frac{A_1 - A_2}{4\pi B_0^2} \cos \frac{\pi}{c} y + \left(\frac{2}{c} - \frac{3(A_1 - A_2)}{4\pi B_0^2} - \frac{3(A_1 - A_4)}{16\pi B_0^2} \right) \cos \frac{3\pi}{c} y + \right.$$

$$\left. + \frac{5(A_1 + A_4)}{16\pi B_0^2} \cos \frac{5\pi}{c} y \right] \exp\left\{ - \left[\left(\frac{3\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{3\pi}{c} \frac{A_3}{2} \right] t \right\} +$$

$$+ \left[\frac{A_2 - A_3}{24\pi B_0^2} \cos \frac{\pi}{c} y + \frac{3(A_1 - A_4)}{16\pi B_0^2} \cos \frac{3\pi}{c} y + \right.$$

$$\left. + \left(\frac{1}{c} - \frac{5(A_2 + A_3)}{24\pi B_0^2} - \frac{5(A_1 + A_4)}{16\pi B_0^2} \right) \cos \frac{5\pi}{c} y \right] \times$$

$$\times \exp\left\{ - \left(\frac{5\pi}{c}\right)^2 \frac{B_0^2}{2} t \right\}, \tag{25}$$

where, in correspondence with section 2, we have discarded the last term in the expressions for the e.v. The probability of no failures up to time t is approximately equal to

$$P(t) = \left[\frac{4}{\pi} - \frac{c(6A_1 - A_3)}{6\pi^2 B_0^2} \right] \exp\left\{ - \left[\frac{B_0^2}{2} \left(\frac{\pi}{c}\right)^2 - \frac{\pi}{c} \frac{A_1}{2} \right] t \right\} +$$

$$+ \left[-\frac{4}{3\pi} + \frac{5cA_1}{4\pi^2 B_0^2} \right] \exp\left\{ - \left[\left(\frac{3\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{3\pi}{c} \frac{A_3}{2} \right] t \right\} +$$

$$+ \left[\frac{2}{5\pi} - \frac{c(3A_1 - 2A_3)}{12\pi^2 B_0^2} \right] \exp\left\{ - \frac{B_0^2}{2} \left(\frac{5\pi}{c}\right)^2 t \right\}. \tag{26}$$

In the course of time, the terms in (25) and (26) with higher e.v. are damped and P(t), for example, tends to the asymptote

$$\lim_{t \to \infty} P(t) = \left[\frac{4}{\pi} - \frac{c(6A_1 - A_3)}{6\pi^2 B_0^2} \right] \exp\left\{ - \left[\left(\frac{\pi}{c}\right)^2 \frac{B_0^2}{2} - \frac{\pi}{c} \frac{A_1}{2} \right] t \right\},$$

shown by the dashed line on Fig. 4.

From (26) and (9) we easily obtain the distribution law for the times of failure. The average time until failure is approximately

$$T \approx \frac{c^2}{4B_0^2} + \frac{2c^3 A_1}{\pi^2 B_0^4}, \tag{27}$$

where the first term equals the average time until failure with no feedback, and the second term shows the increase of this time attributable to the feedback. Relationship (27) can be verified by solving approximately the equation for T as a function of the initial value $y(0) = y_0$ [8]

$$\frac{B_0^2}{2}\frac{d^2T}{dy_0^2} + [V - A(y_0)]\frac{dT}{dy_0} + 1 = 0 \tag{28}$$

for $V = 0$, $cA(y)/B_0^2 \ll 1$ and boundary conditions $T(\pm c/2) = 0$, and by substituting $y_0 = 0$ in the solution.

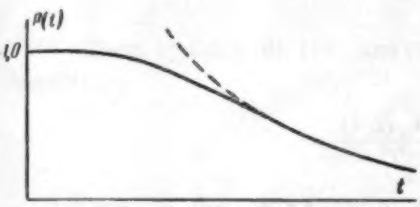

Fig. 4. Probability of no failures, as a function of time.

By substituting the values of B and the A_l in the form of functions of the circuit parameters and the input signal-to-noise ratio (S/N) in (25), (26), and (27), we obtain, for concrete servo systems, the functions W(t,y), P(t), and T expressed in terms of the quantities with which one ordinarily works in analyzing noise stability. By fixing the tracking time, we obtain the dependence of absence of failures on the S/N ratio. The threshold value of the S/N ratio, determined in some way or another, obviously depends on the time interval during which faultless operation is required of the system.

Relationships (25)-(27) become inapplicable only for a significant lowering of the noise level, when failures become improbable. Thus, the S/N ratio threshold is the boundary of the region in which the formulas given here are applicable.

4. More Complicated Cases

The method of analyzing tracking failures given in sections 2 and 3 can be extended to more complicated cases when the diffusion coefficient depends on the error and when the quantity being tracked varies with a constant velocity $x = Vt$ if the basic condition, namely, a large effective passband of the discriminator, is met. With a constant velocity V of the measured quantity, the equation for determining the e.f. and the e.v. takes the form:

$$\frac{B_0^2}{2}Y''(y) + \{(A(y) - V)Y(y)\}' + \lambda Y(y) \equiv L\{Y(y)\} + \lambda Y(y) = 0, \tag{29}$$

where L{ } is a linear operator. For the Ritz-Galerkin method [11], it is necessary that the left member of (29), after the linear combinations of the <u>k</u> coordinate functions are substituted in it, be orthogonal to all these functions. The presence of the velocity introduces an asymmetry in the coordinates, so that the entire set in (14) must be taken. As a result, for example, we have for three coordinate functions, instead of (19), the following equation for the e.v.:

$$\begin{vmatrix} \left(\frac{\pi}{c}\right)^2\frac{B_0^2}{2} - \frac{\pi}{c}\frac{A_1}{2} - \lambda & \frac{8V}{3c} & \frac{\pi}{c}\frac{A_1 - A_2}{2} \\ -\frac{8V}{3c} & \left(\frac{2\pi}{c}\right)^2\frac{B_0^2}{2} + \frac{2\pi}{c}\frac{A_2}{2} - \lambda & -\frac{24V}{5c} \\ -\frac{3\pi}{c}\frac{A_1 + A_2}{2} & \frac{24V}{5c} & \left(\frac{3\pi}{c}\right)^2\frac{B_0^2}{2} - \frac{3\pi}{c}\frac{A_3}{2} - \lambda \end{vmatrix} = 0.$$

By means of this equation, we can again obtain the probability of no failures. We give the expression now for the average time until failure, obtained from Eq. (28):

$$T = \frac{c^2}{4B_0^2} + \frac{2c^3A_1}{\pi^3B_0^4} - \frac{c^4V^2}{48B_0^6} + \frac{c^4A_1^2}{8\pi^3B_0^6}. \tag{30}$$

A comparison of formulas (30) and (27) shows that the mean time until failure decreases when a significant velocity V exists. This is explained by the occurrence of a velocity error which moves the "representative point" to one of the boundaries. For some critical value of velocity, the increase in T due to the feedback is completely compensated by the presence of this velocity, as a result of which any prolonged tracking becomes impossible.

Taking the variability of the diffusion coefficient into account may be done by any of several methods. One of these is a transition to new functions and new coordinates [7]. We introduce the functions

$$\eta(y) = B(\infty)\int_0^y \frac{ds}{B(s)}, \qquad W_1(t, \eta) = \frac{B(y)}{B(\infty)} W(t, y)\big|_{y=y(\eta)},$$

$$A_1(\eta) = \left\{A(y)\frac{B(\infty)}{B(y)} + \frac{B(\infty)B'(y)}{2}\right\}\bigg|_{y=y(\eta)}.$$

We can then show that

$$\frac{\partial W_1}{\partial t} = \frac{\partial}{\partial \eta}\{A_1(\eta) W_1(t, \eta)\} + \frac{B^2(\infty)}{2}\frac{\partial^2 W_1(t, \eta)}{\partial \eta^2}.$$

We thus arrive at a previous equation, whose method of solution on the interval $[\eta(-c/2), \eta(+c/2)]$ is known.

Another method is the direct application of the Ritz-Galerkin method. The difference from the case of section 3 will consist of the appearance of the harmonic function $B^2(y)$, expanded in a Fourier series on the interval $[-c/2, +c/2]$.

5. Tracking Failures in AFC Systems

An AFC system is an example of a servo system with a nonlinear discriminator for which the analysis given above is valid. The block schematic of an AFC is given in Fig. 5. In this case, the mixer, the band amplifier and the frequency discriminator properly so-called correspond to the discriminator in the circuit of Fig. 1, while, for the second part, there correspond the smoothing circuit, the dc amplifier, the frequency control devices, and the heterodyne, $u_1(t)$ and $u_3(t)$ are the voltages applied to the mixer, $u_2(t)$ is the output voltage of the frequency discriminator, $z(t)$ is the heterodyne frequency minus (plus) the mean frequency of the band amplifier, $y(t)$ is the current difference (error) of the average frequencies of the heterodyne signal and the amplifier. The operations used to obtain $u_2(t)$ from $u_1(t)$ and $u_3(t)$ include heterodyning, amplification, limiting, new (separate) amplification by the two tuned circuits, amplitude detection, and subtraction of the results of detection.

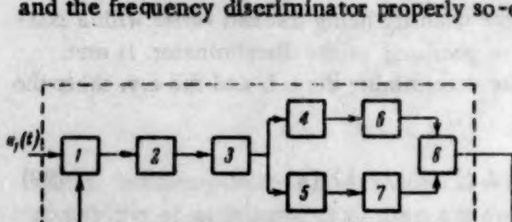

Fig. 5. Block schematic of an AFC: 1) mixer; 2) band amplifier; 3) limiter; 4,5) tuned circuits; 6,7) amplitude detectors; 8) subtraction circuit; 9) smoothing circuit; 10) frequency control device; 11) heterodyne.

For the spectral densities of the noise and the random signal at the input we take, respectively, S_n = const and $S_s = \exp\{-(\omega-\omega_s)^2/\Delta_s^2\}$. With the dimensionless frequency characteristic of the band amplifier equal to $\exp\{-(\omega-\omega_0)^2/2\beta^2\}$, the impulsive response of the tuned circuits equal to $\omega_0 e^{-\alpha t}\cos(\omega_0 \pm \omega_1)t$, with a square-law detector and for $\alpha, \omega_1, \Delta_s, \beta \ll \omega_0, \omega_s$, we determine the discrimination characteristic $a\{y\}$ and the output spectral density $S\{y\}$. For $S_n/S_s = \kappa > 1$, we get the expressions

$$a\{y\} \approx \frac{\sqrt{\pi}\, k_1 \omega_0^2}{4\alpha\Delta_n \times \sqrt{1+\mu^2}} e^{-\frac{y^2}{(1+\mu^2)\Delta_c^2}} \operatorname{Re}\left[w\left(\frac{ky-\omega_1}{\Delta_n} + i\frac{\alpha}{\Delta_n}\right) - w\left(\frac{ky+\omega_1}{\Delta_n} + i\frac{\alpha}{\Delta_n}\right)\right], \tag{31}$$

$$S\{y\} \approx \frac{\pi k_1^2 \omega_0^4}{4\alpha^3\beta^2}\left\{e^{-\frac{2\omega_1^2}{\beta^2}}\left(1 - \frac{\sqrt{2\pi}\alpha}{\beta} w\left(i\frac{\sqrt{2}\,\alpha}{\beta}\right)\right) + \right.$$
$$\left. + \frac{\sqrt{2\pi}\alpha}{\beta}\operatorname{Re}\left[w\left(\frac{\omega_1 + i\alpha}{\beta/\sqrt{2}}\right)\right] - \frac{\alpha^2}{\alpha^2+\omega_1^2}\right\}, \tag{32}$$

where $\mu=\frac{\beta}{\Delta_c}$, $\Delta_n^2=\frac{\beta^2\Delta_c^2}{\beta^2+\Delta_c^2}$, $k=\frac{\mu^2}{1+\mu^2}$, k_1 is a proportionality factor and $w(z)$ is the probability integral with a complex argument:

$$w(z)=\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-t^2+2izt}\,dt.$$

We now determine the coordinates of the "absorbing barrier" in terms of the effective width of the selected domain. From (31) we have that

$$c=\int_{-\infty}^{+\infty}\exp\left\{-\frac{y^2}{(1+\mu^2)\Delta_c^2}\right\}dy=\sqrt{\pi(1+\mu^2)}\,\Delta_c.$$

Then, by using (6), (18), and (31), we can obtain the following approximate values for the Fourier coefficients:

$$A_l=\frac{2\sqrt{\pi}K_v k_1\omega_0^2}{\alpha\beta\varkappa\sqrt{1+\mu^2}}\exp\left\{-\frac{\pi l^2}{1+\mu^2}-\right.$$
$$\left.-2\sqrt{\frac{\pi}{1+\mu^2}}\frac{\alpha}{\Delta_c}l-\frac{\omega_1^2-\alpha^2}{\beta^2}\right\}\sin\frac{2\omega_1\left[l\sqrt{\pi}k\beta-\alpha\right]}{\beta^2}. \quad (33)$$

Substitution of (32) and (33) in (25), (24), or (27) leads to the sought-for probability functions. For concrete values of the parameters, all necessary characteristics of tracking failures can be obtained from these formulas.

SUMMARY

Random processes in nonlinear servo systems in which the element for isolating the error signal has sufficiently low inertia in comparison with the inertia of the closed-loop system, can be conveniently investigated by means of diffusion equations. With a significantly high noise level, the conditions for the applicability of this apparatus are generally met, due to the decrease in the system's gain. In this work, we obtained formulas for the probability of failure of tracking under the effect of fluctuating noise for any form of nonlinearity of the discrimination characteristic.

The development of these methods, as applied to problems of automatic control, begun in [9], is apparently fruitful, and can be carried out in several directions. It is of interest to consider systems with high astatism. The solution of such problems would, in many important practical cases, deliver one from the necessity of using numerical methods for the statistical calculation of servo systems containing nonlinear elements.

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ON THE COMPUTATION OF THE MEAN SQUARE ERROR OF PROCESSING A STATIONARY RANDOM SIGNAL BY A LINEAR AUTOMATIC CONTROL SYSTEM

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A formula is presented for the approximate computation of a convolution integral (a Duhamel integral) which has a high degree of accuracy for a large integration step.

A method is given for the approximate solution (evaluation) of a double-convolution integral which permits the rapid computation of the magnitude of the mean square error of processing a stationary random signal by an automatic control system.

The relationship between the correlation functions of the input stimulus and the error of a linear automatic control system can be presented in the form of some double-convolution integrals [1, 2]. Consequently, the problem of computing the correlation function of the error of a linear system leads to the computation of the double-convolution integral obtained. An exact solution of a convolution integral is an arduous task, and is not always possible in practice.

In automatic control theory, a great deal of attention has been given, and is being given, to the development of methods for the approximate computation of convolution integrals. In this direction, particularly great interest inheres in the results obtained by A.A. Krasovskii, G.S. Pospelov, Ya.Z. Tsypkin, and O.B. Lur'e.

The method of approximate computation of convolution integrals presented in this paper is based, fundamentally, on the results obtained in [3, 4].

The high accuracy in computing convolution integrals by the method given here, using a large integration step, allows one to use this method in computating the aforementioned double-convolution integrals.

The ordinary graphicoanalytic method of computation [1, 2], based on the Fourier transformation of the double-convolution integral, is more arduous and less accurate.

Approximate Method for Computing a Convolution Integral (Duhamel Integral)

If $f(t)$ is the stimulus acting on the system and $k(t)$ is the impulsive-response function, then the reaction $x(t)$ of the system to stimulus $f(t)$ is expressed by the integral

$$x(t) = \int_0^t f(\tau)\, k(t-\tau)\, d\tau, \tag{1}$$

where $f(\tau) = 0$ for $\tau \le 0$.

Or, going to the Laplace transforms, we obtain

$$X(p) = F(p)K(p), \quad (2)$$

where F(p) is the Laplace transform of the stimulus $f(t)$, K(p) is the Laplace transform of the impulsive response k(t) or, for zero initial conditions, the system's transfer function and X(p) is the Laplace transform of the output function x(t). Obviously, the stimulus function $f(t)$ can be the output of a link or a system of automatic control.

Fig. 1.

We replace the graph of the continuous function $f(t)$ (Fig. 1) by a broken-line curve which coincides with the continuous curve at points 1, 2, 3, . . . [3, 4]. This broken-line curve can be given in the form of isosceles triangles with bases equal to 2τ and with heights equal to the ordinates of the continuous function at the points 1, 2, 3, . . . , shifted with respect to the origin of coordinates by τ, 2τ, 3τ, . . . We call pulses of this form Δ-functions.

We denote the representation of an elementary pulse with unit amplitude by h(p). The quantity

$$h(p) = \frac{(1 - e^{-p\tau})^2}{\tau p^2} e^{\tau p}$$

we call a unit Δ-function.

According to the theorem on lags, the representation of the elementary pulse of height $f(i\tau)$ at point $t = i\tau$ will equal $f(i\tau)h(p)e^{-i\tau p}$.

Consequently, the representation for the piecewise linear approximation to function $f(t)$ can be written in the following way:

$$F(p) \approx h(p) \sum_{i=0}^{\infty} f(i\tau)\, e^{-i\tau p}.$$

From whence expression (2) can be rewritten in the form

$$X(p) \approx h(p)\, K(p) \sum_{i=0}^{l} f(i\tau)\, e^{-pi\tau} \quad (3)$$

for

$$X(p) \approx f(\tau)\, h(p)\, K(p)\, e^{-p\tau} + f(2\tau)\, h(p)\, K(p)\, e^{-2\tau p} + \ldots$$
$$\ldots + f(i\tau)\, h(p)\, K(p)\, e^{-i\tau p} + \ldots + f(l\tau)\, h(p)\, K(p)\, e^{-l\tau p}.$$

By turning to the inverse Laplace transforms, we obtain

$$x(t) \approx f(\tau)\, k_\Delta(t - \tau) + f(2\tau)\, k_\Delta(t - 2\tau) + \ldots$$
$$\ldots + f(i\tau)\, k_\Delta(t - i\tau) + \ldots + f(l\tau)\, k_\Delta(t - l\tau), \quad (4)$$

where $k_\Delta(t)$ is the transient response in the system when a unit Δ-function acts on its input and $f(i\tau)k_\Delta(t-i\tau)$ is the transient response in the system when a Δ-function of height $f(i\tau)$, shifted by $i\tau$ with respect to the origin of coordinates, acts on its input. The values of the ordinates at the points which are multiples of τ are defined by the expressions

$$x(l\tau) \approx f(\tau)\, k_\Delta(0) + f(\tau)\, k_\Delta(2\tau - \tau) + f(\tau)\, k_\Delta(3\tau - \tau) + \ldots$$
$$\ldots + f(2\tau)\, k_\Delta(0) + f(2\tau)\, k_\Delta(2\tau - \tau) + f(2\tau)\, k_\Delta(3\tau - \tau) + \ldots \quad (5)$$

With a different grouping of terms in expressions (5) we obtain the following formulas:

$$x(lt) = \sum_{i=0}^{l} f(i\tau)\,k_\Delta(l\tau - i\tau) \qquad (6)$$

$$\begin{pmatrix} i = 0, 1, 2, \\ j = 0, 1, 2, \dots \end{pmatrix},$$

$$\widetilde{x(l\tau)} \approx \widetilde{f(i\tau)}\,\widetilde{k_\Delta(j\tau)} \qquad (7)$$

where $\widetilde{f(i\tau)}$ and $\widetilde{k_\Delta(j\tau)}$ are numerical sequences which are multiplied by the rules governing polynomials.*

Formulas (6) and (7) give approximate values of the integral in (1). In these formulas, the ordinates $k_\Delta(j\tau)$ are unknown. These ordinates can be found from the formulas $k_\Delta(t) = L^{-1}h(p)K(p)$ or

$$k_\Delta(t) \approx L^{-1}\frac{K(p)\,e^{\tau p}}{\tau p^2} - 2L^{-1}\frac{K(p)}{\tau p^2} + L^{-1}\frac{K(p)}{\tau p^2}e^{-\tau p}, \qquad (8)$$

where τ is the integration step.

The size of the integration step is determined from the condition that the approximation of the curve for $f(t)$ by the Δ-functions be sufficiently accurate. For practical computations, it is sometimes more convenient to use the formula

$$\widetilde{k_\Delta(j\tau)} = \widetilde{k_0(j\tau + \tau)}\,(0.5; 0; -0.5) - k_0(0 + \tau)\,(0.5; -0.5), \qquad (9)$$

where $k_0(t)$ is the system's transient response (to a unit function at its input).

If $k_0(0) = 0$ for $t = 0$, then formula (9) takes the form:

$$\widetilde{k_\Delta(j\tau)} = \widetilde{k_0(j\tau + \tau)}\,(0.5; 0; -0.5). \qquad (10)$$

These formulas are the inverse Laplace transforms of the following expression [3]:

$$K_\Delta(q) = K_0(q)\frac{1 - e^{-2\tau p}}{2e^{-\tau p}} - \frac{k_0(0)(1 - e^{-\tau p})}{2e^{-\tau p}}, \qquad (11)$$

where $K_0(q)$ is the discrete Laplace transform of the transient response of an automatic control system when a unit function is applied to its input and $K_\Delta(q)$ is the discrete Laplace transform of the transient response in the system when a unit Δ-function is applied to its input.

Estimate of the Error in Computing a Convolution Integral by the Approximate Formulas

Formally, the approximate solution obtained by the use of the formulas given above coincides with the ordinary approximate computation of a convolution integral (or Duhamel integral). In practice, the ordinary approximate expressions for the convolution integral are almost never used, which is explained by the low accuracy of this method when a finite interval τ is chosen. If the interval τ is decreased, then the accuracy of the computation of the convolution integral increases, but there is a corresponding increase in the amount of computational work.

It can be shown that the computation of a convolution integral by the ordinary formulas gives a significantly greater error than its computation by our method. This is due to two factors.

The effect of the first factor consists of the following. If the stimulus function is presented in the form of an arbitrarily chosen transient response (Fig. 2) then, to solve the convolution integral by the ordinary approximation formulas, the ordinates are taken at points which are multiples of τ. By multiplying these ordinates by τ we obtain the hatched area on Fig. 2 which is bounded by the broken lines. Both the area bounded by this curve and the form of the curve are approximated very roughly. With the other method, the one used by us (Fig. 3), the approximation is effected by means of the triangles very accurately for the same interval τ.

***A wavy line above indicates a numerical sequence.**

The essence of the second methodical error is explained as follows. In computing the transient response by the ordinary approximate expression for the convolution integral, the system's transient response is taken for the stimulus by a pulse function, but the approximation of the stimulus is done by means of rectangles. With very small intervals τ, the transient response for a pulse stimulus and the transient response for a stimulus of rectangular shape and finite duration, with a unit area, will coincide for all practical purposes.

The greater is τ, the more significant will be the difference between the transient response obtained from a pulse stimulus and that obtained from a rectangular-shaped stimulus and, consequently, the greater will be the error in computing a convolution integral by the ordinary approximate method.

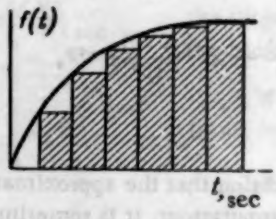

Fig. 2.

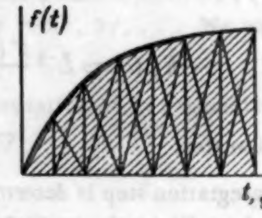

Fig. 3.

In the method herein presented, we choose for our ordinates, not a pulse transient response of the system, but the system's transient response to unit Δ-functions at its input, as a result of which the analogous error does not occur.

In computing the convolution integral by the approximate formulas given in [3], the first methodical error is just about equal to the first methodical error obtained with the method presented here. In contradistinction to the method presented here, the second methodical error appears in full force when the earlier formulas are used.

In computing convolution integrals by the approximate formula given in [5], the second methodical error is eliminated, but the first remains in its entirety.

It is clear from what has been said that the accuracy of computation of convolution integrals by the approximation formulas given here will, for large intervals τ, be much higher than the computational accuracy of the approximation formulas cited. This permits the approximation formulas given here to be used in practical computations for automatic control systems.

Computation of Mean Square Errors

We assume that a stationary random function $m(t)$ is applied to the input of a linear automatic control system (Fig. 4), where $k(t)$ is the system's impulsive error response. The system's task is to process this random function with some mean square error.

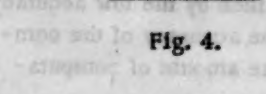

Fig. 4.

We denote the correlation function of the input signal by $R_m(\tau)$. The mean square value of the function $m(t)$ will equal $R_m(0)$. The correlation function of the system's error will be defined by the expression [1]

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} d\lambda \int_{-\infty}^{\infty} k(\eta)\, k(\lambda)\, R_m(\tau + \lambda - \eta)\, d\eta. \tag{12}$$

We make the change of variables:

$$\tau + \lambda - \eta = \xi, \quad \eta = \tau + \lambda - \xi. \tag{13}$$

We then find the new limits of integration:

$$\eta = -\infty, \quad \xi = \tau + \lambda + \infty = \infty,$$
$$\eta = \infty, \quad \xi = \tau + \lambda - \infty = -\infty. \tag{14}$$

Expression (12) is transformed to the form

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} k(\lambda)\, d\lambda \int_{-\infty}^{\infty} R_m(\xi)\, k(\tau + \lambda - \xi)\, d\xi. \tag{15}$$

By changing the order of integration we get

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} R_m(\xi) d\xi \int_{-\infty}^{\infty} k(\lambda)\, k(\tau + \lambda - \xi)\, d\lambda. \tag{16}$$

We introduce the new variables:

$$\tau + \lambda - \xi = \theta, \quad \lambda = \theta - \tau + \xi. \tag{17}$$

We again find the new limits of integration:

$$\lambda = -\infty, \quad \theta = \tau - \xi - \infty = -\infty,$$
$$\lambda = \infty, \quad \theta = \tau - \xi + \infty = \infty.$$

With the introduction of the new variable θ, expression (16) is transformed to the form

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} R_m(\xi)\, d\xi \int_{-\infty}^{\infty} k(\theta)\, k[-(\tau - \theta - \xi)]\, d\theta. \tag{18}$$

We now consider the first integral:

$$k_x(\tau - \xi) = \int_{-\infty}^{\infty} k(\theta)\, k[-(\tau - \theta - \xi)]\, d\theta.$$

The function $k(\tau-\theta-\xi)$ differs from the function $k[-(\tau-\theta-\xi)]$ only by the sign of the argument. It is well known that the impulsive response of any stable system must satisfy the condition

$$\int_{0}^{\infty} |k(\tau)|\, d\tau < \infty.$$

Moreover, from the condition of physical realizability, we get

$$k(\theta) = 0 \quad \text{for } \theta < 0,$$
$$k[-(\tau - \theta - \xi)] = 0 \quad \text{for } \theta < \tau - \xi.$$

With the conditions of physical realizability taken into account, the first integral of expression (18) assumes the form:

$$k_x(\tau - \xi) = \int_{\tau-\xi}^{\infty} k(\theta)\, k[-(\tau - \xi - \theta)]\, d\theta.$$

The approximate expression for the first integral will be

$$\int_{\tau-\xi}^{\infty} k(\theta)\,k[-(\tau-\theta-\xi)]\,d\theta = \int_{\vartheta}^{\infty} k(\theta)\,k[-(\vartheta-\theta)]\,d\theta \approx$$

$$\approx \sum_{i=j}^{i=l} k(i\theta_1)\,k_\Delta[-(l\vartheta_1 - i\theta_1)], \tag{19}$$

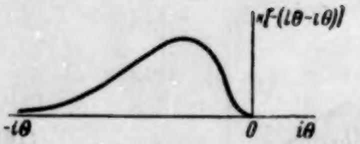

Fig. 5.

where $\vartheta = \tau - \xi$, $\vartheta_1 = \theta_1$, θ_1 is the time interval chosen, $k_\Delta[-(l\theta_1 - i\theta_1)]$ is the ordinate of the system's transient response when a unit Δ-function is applied to its input [this transient response coincides with the transient response $k_\Delta(l\theta_1 - i\theta_1)$ with the difference that the argument is a negative quantity (Fig. 5)], $k(i\theta_1)$ is the ordinate of the Δ-function by which the pulse transient response $k(t)$ is approximated.

The second integral equals

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} R_m(\xi)\,k_x(\tau-\xi)\,d\xi. \tag{20}$$

In formula (20), $R_m(\xi)$ and $k_x(\tau-\xi)$ are even functions (Figs. 6 and 7). We present the function $k_x(\tau-\xi)$ as the sum of two functions $k_x(\tau-\xi) = k_{x_1}(\tau-\xi) + k_{x_1}[-(\tau-\xi)]$,

where $k_{x_1}(\tau-\xi) = 0$ for $\tau-\xi < 0$, $k_{x_1}[-(\tau-\xi)] = 0$ for $\tau-\xi > 0$.

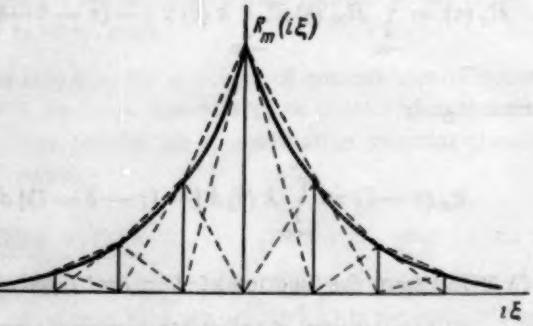

Fig. 6.

Then, integral (20) can be given by the expression*

$$R_\varepsilon(\tau) = \int_{-\infty}^{\infty} R_m(\xi)\,\{k_{x_1}(\tau-\xi) + k_{x_1}[-(\tau-\xi)]\}\,d\xi =$$

$$= \int_{-\infty}^{\tau} R_m(\xi)\,k_{x_1}(\tau-\xi)\,d\xi + \int_{\tau}^{\infty} R_m(\xi)\,k_{x_1}[-(\tau-\xi)]\,d\xi. \tag{21}$$

$$\int_{\tau}^{\infty} R_m(\xi)\,k_{x_1}(\tau-\xi)\,d\xi = 0, \text{ since } \quad k_x(\tau-\xi) = 0 \text{ for } \tau < \xi,$$

$$\int_{-\infty}^{\tau} R_m(\xi)\,k_{x_1}[-(\tau-\xi)]\,d\xi = 0, \text{ since } \quad k_x[-(\tau-\xi)] = 0 \text{ for } \quad \tau > \xi.$$

By using the approximation formulas, we can present expression (21) in the form

$$R_\varepsilon(j\xi_1) = \sum_{i=-l}^{i=j} R_m(i\xi_1)\, k_{x_1\Delta}(j\xi_1 - i\xi_1) + \sum_{i=j}^{i=l} R_m(i\xi_1)\, k_{x_1\Delta}[-(j\xi_1 - i\xi_1)]\,, (22) \tag{22}$$

where $R_m(i\xi_1)$ are the ordinates of the Δ-functions by which the correlation function is approximated.

To determine the $k_{x_1\Delta}(j\xi_1 - i\xi_1)$ and the $k_{x_1\Delta}[-(j\xi_1 - i\xi_1)]$, it is necessary first to compute the $k_{x\Delta}(j\xi_1 - i\xi_1)$ by formula (19) where, instead of $k(i\theta_1)$, it is necessary to substitute $k_\Delta(i\theta_1)$, and thereafter to present $k_{x\Delta}(j\xi_1 - i\xi_1)$ as the sum of two functions, as was stated above.

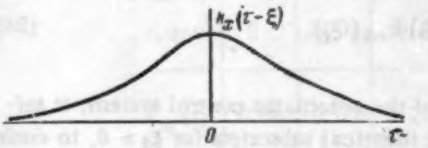

Fig. 7.

The values of the ordinates $k_\Delta(i\theta_1)$ of the transient response when unit Δ-functions act on the input of the system are found from formulas (9), (10), or from formula (8).

If the interval ξ_1 is so chosen that the continuous function of time $k_0(t)$ can, with sufficient accuracy, be approximated by a broken line, then formulas (9), (10) will be sufficiently accurate.

The ordinates of the functions $k_{x_1\Delta}(j\xi_1 - i\xi_1)$ and $k_{x_1\Delta}[-(j\xi_1 - i\xi_1)]$ will equal, respectively,

$$\frac{k_{x\Delta}(0)}{2},\quad k_{x\Delta}(\xi_1),\quad k_{x\Delta}(2\xi_1),\quad k_{x\Delta}(2\xi_1),\ldots,\quad k_{x\Delta}[(j-1)\,\xi_1],\quad k_{x\Delta}(j\xi_1);$$

$$\frac{k_{x\Delta}(0)}{2},\quad k_{x\Delta}[-(-\xi_1)],\quad k_{x\Delta}[-(-2\xi_1)],\quad k_{x\Delta}[-(-2\xi_1)],\ldots,$$
$$k_{x\Delta}[-(-j+1)\,\xi_1],\quad k_{x\Delta}[-(-j\xi_1)].$$

As was stated earlier, the function $k_{x\Delta}(j\xi_1 - i\xi_1)$ is even, and the ordinates

$$k_{x\Delta}(\xi_1)\,\text{and}\,k_{x\Delta}[-(-\xi_1)],\quad k_{x\Delta}[-(2\xi_1)]\,\text{and}\,k_{x\Delta}[-(-2\xi_1)],$$
$$k_{x\Delta}(3\xi_1)\,\text{and}\,k_{x\Delta}[-(-3\xi_1)]$$

are, respectively, equal among themselves.

Therefore, to compute the correlation function $R_\varepsilon(\tau)$, one can compute one of the two integral sums given in formula (22). For example, $R_\varepsilon(j\xi_1) = \sum_{i=-l}^{j} R_m(i\xi_1)\, k_{x\Delta}(j\xi_1 - i\xi_1)$. The second integral sum is obtained from the computed one by replacing ξ_1 by $-\xi_1$.

By adding the ordinates of the function thus computed, we obtain the ordinates of the error correlation function of the automatic control system.

By using formula (22), we compute the value of the correlation function's ordinate for $i\xi_1 = 0$:

$$R_{\varepsilon_1}(0) = \sum_{i=-l}^{i=j=0} R_m(i\xi_1)\, k_{x\Delta}(0\xi_1 - i\xi_1) =$$

$$= R_m(-l\xi_1)\, k_{x\Delta}(l\xi_1) + R_m(-l\xi_1 + \xi_1)\, k_{x\Delta}(l\xi_1 - \xi_1) +$$
$$+ R_m(-l\xi_1 + 2\xi_1)\, k_{x\Delta}(l\xi_1 - 2\xi_1) + \ldots + R_m(-\xi_1)\, k_{x\Delta}(\xi_1) + R_m(0)\,\frac{k_{x\Delta}(0)}{2},$$

$$R_{\varepsilon_2}(0) = \sum_{i=j=0}^{i=l} R_m(i\xi_1)\, k_{x\Delta}[-(0\xi_1 - i\xi_1)] =$$

$$= R_m(l\xi_1)\,k_{x\Delta}[-(-l\xi_1)] + R_m(l\xi_1-\xi_1)\,k_{x\Delta}[-(-l\xi_1+\xi_1)] +$$
$$+ R_m(l\xi_1-2\xi_1)\,k_{x\Delta}[-(-l\xi_1+2\xi_1)] + \ldots$$
$$\ldots + R_m(\xi_1)\,k_{x\Delta}[-(-\xi_1)] + R_m(0)\,\frac{k_{x\Delta}(0)}{2}.$$

Since $R_m(l\xi_1) = R_m(-l\xi_1)$ and $k_{x\Delta}(l\xi_1) = k_{x\Delta}[-(-l\xi_1)]$, then

$$\begin{gathered} R_\varepsilon(0) = R_{\varepsilon_1}(0) + R_{\varepsilon_2}(0) = 2R_m(-l\xi_1)\,k_{x\Delta}(l\xi_1) + \\ + 2R_m(-l\xi_1+\xi_1)\,k_{x\Delta}(l\xi_1-\xi_1) + 2R_m(-l\xi_1+2\xi_1)\,k_{x\Delta}(l\xi_1-2\xi_1) + \ldots \\ \ldots + 2R_m(-\xi_1)\,k_{x\Delta}(\xi_1) + R_m(0)\,k_{x\Delta}(0) \end{gathered} \tag{23}$$

or

$$R_\varepsilon(0) = R_m(0)\,k_{x\Delta}(0) + 2\sum_{i=1}^{i=l} R_m(i\xi_1)\,k_{x\Delta}(i\xi_1). \tag{24}$$

Thus, to obtain the null ordinate of the error correlation function of the automatic control system, it suffices to multiply the ordinates of the functions $R_m(i\xi_1)$ and $k_{x\Delta}(i\xi_1)$ with identical subscripts for $\xi_1 > 0$, to double the product thus obtained (except for the product of the null ordinates) and to add them. In certain cases, formula (24) can be replaced by a simpler expression (cf. the Appendix).

The mean square value of the error is found from the expression $\epsilon = \sqrt{R_\varepsilon(0)}$.

APPENDIX

Computation of the Mean-Square Error

Using our approximate method, we now compute the mean-square error of an automatic control system for a radar set with a random stationary noise function at its input. In the given case, $\epsilon = \sqrt{R_\varepsilon(0)} = \sqrt{R_x(0)}$. The data which characterize the properties of this system are given in [2].

The system's open-loop transfer function has the form:

$$W(p) = \frac{80\,(0.36p+1)}{p\,(16p+1)}.$$

The closed-loop transfer function is

$$K(p) = \frac{1.8p+5}{p^2+1.86p+5}. \tag{25}$$

The generalized transfer function of the system when a unit function is applied to its input is defined by the expression

$$K_0(p) = \frac{1.8p+5}{p\,(p^2+1.86p+5)},$$

and the transient response can be presented as

$$k_0(t) = 1 - 1.084e^{-0.93t}\cos(2.04t+0.4).$$

The curve of the transient response is given in Fig. 8. This curve can be approximated sufficiently well by straight-line segments chosen with an interval of $\xi = 0.4$.

The ordinates of the transient response, taken at intervals of $\xi_1 = 0.4$, are given in Table 1.

TABLE 1

t, sec.	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4
$k_0(t)$	0	0.770	1.216	1.310	1.189	0.966	0.945	0.933	0.965	0.997	1.013	1.012

We observe in formula 10 the ordinates of the transient-response process for reaction of a single input Δ function.

$0.5k_0(j\tau+\tau)$	0.385	0.608	0.655	0.595	0.483	0.473	0.467	0.483	0.499	0.507	0.506
$0.5k_0(j\tau+\tau)$			−0.385	−0.608	−0.655	−0.595	−0.483	−0.473	−0.467	−0.483	−0.499

By adding the columns of this table, we obtain the values of the ordinates of $k_\Delta(t)$ (cf. Table 2).

TABLE 2

t, sec	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0
$k_\Delta(t)$	0.385	0.608	0.270	−0.013	−0.172	−0.122	−0.016	0.010	0.032	0.024	0.007

We now find the ordinates of the "nominal" system when a unit Δ-function acts on its input, i.e., we compute the ordinates by the formula

$$\sum_{i=j}^{l} k_\Delta(i\xi_1)\, k_\Delta[-(l\xi_1 - i\xi_1)] = \widetilde{k_\Delta(i\xi_1)\, k_\Delta[-(l\xi_1 - i\xi_1)]}.$$

By multiplying the numerical sequences obtained in accordance with the rules governing polynomials, we obtain the even functions $k_{x\Delta}(l\xi_1)$. By virtue of the symmetry of these functions, it suffices to compute only the ordinates lying to the left (or only to the right) of the axis of ordinates.

The multiplication process is given in Table 3.

TABLE 3

$i\xi_1$	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0
$-(l\xi_1 - i\xi_1)$	−4	−3.6	−3.2	−2.8	−2.4	−2.0	−1.6	−1.2	−0.8	−0.4	0
$k_\Delta(i\xi_1)$	0.385	0.608	0.270	−0.013	−0.172	−0.122	−0.016	0.010	0.032	0.024	0.007
$k_\Delta[-(l\xi_1 - i\xi_1)]$	0.007	0.024	0.032	0.010	−0.016	−0.122	−0.172	−0.013	0.270	0.608	0.385
	0.0030	0.0045	0.0020	−0.0001	−0.0013	−0.0010	−0.0001	0.0001	0.0003	0.0018	0.0001
		0.0093	0.0145	0.0065	−0.0003	−0.0043	−0.0030	−0.0005	0.0003	0.0008	0.0005
			0.0123	0.0195	0.0088	−0.0005	−0.0053	−0.0040	−0.0005	0.0003	0.0010
				0.0038	0.0060	0.0028	−0.0001	−0.0018	−0.0013	−0.0002	0.0001
					−0.0063	−0.0103	−0.0045	0.0023	0.0028	0.0020	0.0003
						−0.0470	−0.0740	−0.0330	0.0018	0.0210	0.0150
							−0.0663	−0.1043	−0.0465	0.0023	0.0295
								−0.0053	−0.0083	−0.0035	0.0002
									0.1038	0.1638	0.0728
										0.2340	0.3695
											0.1455

By summing the columns of Table 3, we obtain the values of the ordinates $k_{x\Delta}(i\xi_1)$:

$i\xi_1$	−4	−3.6	−3.2	−2.8	−2.4	−2.0	−1.6	−1.2	−0.8	−0.4	0
$k_{x\Delta}(i\xi_1)$	0.003	0.014	0.029	0.030	0.007	−0.060	−0.153	−0.148	0.052	0.422	0.637

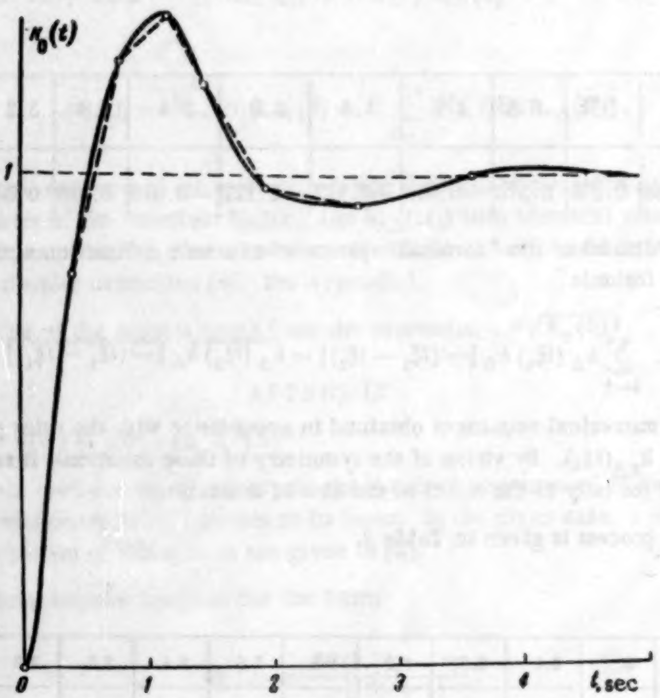

Fig. 8.

Figure 9 shows the graphs of $k_{x\Delta}(i\xi_1)$ (the right side).

In order that formula (24) might be used for computing $R_\varepsilon(0)$, it is necessary that the correlation function of the noise at the system's input be computed.

The normalized autocorrelation function of the input signal (noise) is expressed approximately in the form

$$R_1(\tau) = e^{-24|\tau|}\cos 40\tau.$$

The normalized autocorrelation function of the commutated signal will have the form:

$$R_m(\tau) = 2\cos\omega_s\tau R_1(\tau),$$

where ω_s is the scanning frequency,

$$R_m(\tau) = 2e^{-24|\tau|}\cos 188\,\tau\cos 40\tau. \tag{26}$$

The ordinates of the autocorrelation function, computed by formula (26), are given in Table 4.

With a sufficient degree of accuracy, the function $R_m(\tau)$ can be approximated by line segments with $\xi_1 = 0.005$.

We note that the duration of correlation function $R_m(\tau)$ is much less than the duration of correlation function $k_{x\Delta}(i\xi_1)$. Correlation function $k_{x\Delta}(i\xi_1)$ is approximated sufficiently well by a Δ-function of width $2\xi_1 = 0.8$

TABLE 4

τ	0	0.005	0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050	0.055	0.060
$R_m(\tau)$	2.00	1.03	−0.43	−1.03	−0.70	0.00	0.28	0.14	0.00	0.10	0.24	0.10	−0.10
τ	0.065	0.070	0.075	0.080	0.085	0.090	0.095	0.100	0.105	0.110	0.115	0.120	
$R_m(\tau)$	−0.32	−0.28	−0.02	0.22	0.22	0.08	−0.12	−0.12	−0.02				

sec, but $R_m(\tau) = 0$ for $\tau > 0.4$ and, consequently, correlation function $R_m(\tau)$ can be replaced by a Δ-function of width $2\xi_1 = 0.8$ sec with an area equal to the area bounded by the $R_m(\tau)$ curve and the axis of abscissas.

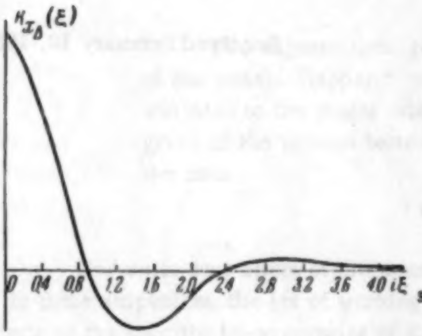

Fig. 9.

This area is $S_{R_m}(\tau) = 2(1 + 1.03 - 0.43 - 1.03 + \ldots + 0.22 + 0.08 - 0.12 - 0.12 - 0.02)\,0.005 = 0.0027$.

The height of the Δ-function of the same area and with width $2\xi = 0.8$ will be equal to $h = 0.00675$.

The value of the ordinate $R_\varepsilon(0)$ is found from the expression

$$R_\varepsilon(0) = hk_{x\Delta}(0) = 0.0043.$$

The actual value is $R_m(0) = 212 \cdot 10^{-6}$ radians². Consequently,

$$R_\varepsilon(0) = 0.91 \cdot 10^{-6}.$$

The mean square value of this error will equal

$$\varepsilon = \sqrt{R_\varepsilon(0)} = 0.975 \cdot 10^{-3} \text{ rad.}$$

The experimental value of the mean square error is

$$\varepsilon_{\text{exp.}} = 1.04 \cdot 10^{-3} \text{ rad.}$$

As is clear from these results, the accuracy in computing the mean square error is high while the difficulty is small (if it is considered that the correlation function of the input signal is known).

If the duration of the correlation functions $R_m(\tau)$ and $k_{x\Delta}(i\xi_1)$ are commensurable, then formula (24) should be used to compute the mean square value $R_m(0)$, which does not increase the difficulty of the computation very much.

If the duration of correlation function $k_{x\Delta}(i\xi_1)$ is much less than the duration of correlation function $R_m(\tau)$, then correlation function $k_{x\Delta}(i\xi_1)$ should be replaced by a Δ-function in the same way as described above. In this case, the correlation function at the system's output will approximately coincide with the input correlation function.

SUMMARY

1. The approximate formula for computing a convolution integral has a high degree of accuracy for a large integration step, and permits the transient response of a linear automatic control system to be determined when an arbitrary function of time, given either analytically or graphically, is impressed on its input.

2. The computation of the mean square error of processing a stationary random signal by a linear automatic control system leads to the solution (evaluation) of some double-convolution integrals.

The approximate formula given here for computing convolution integrals allows this given double convolution integral to be solved with a high degree of accuracy with a small amount of computational work.

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JET POWER EFFECT IN NOZZLE-FLAPPER HYDRAULIC AMPLIFIERS

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The experimental results are given for an investigation of a hydraulic amplifier of the nozzle-flapper type for various combinations of its parameters. Basic attention was paid to the power interaction of the jet and the flapper. A short description is given of the objects tested, the program, the setup, and the methods of carrying out the tests.

Hydraulic amplifiers of the nozzle-flapper type have found wide usage in the devices of automation [1]. In these amplifiers, the jet of working fluid flowing from the nozzle has a power effect on the flapper, which is one of the specific idiosyncrasies of the amplifiers' operation. For choosing the controlling elements for nozzle-flapper amplifiers and for carrying out static computations and analyses of the dynamics of systems which contain one or several such amplifier stages, it is necessary to have numerical data on the magnitude of the forces on the flappers around which the fluid jets flow.

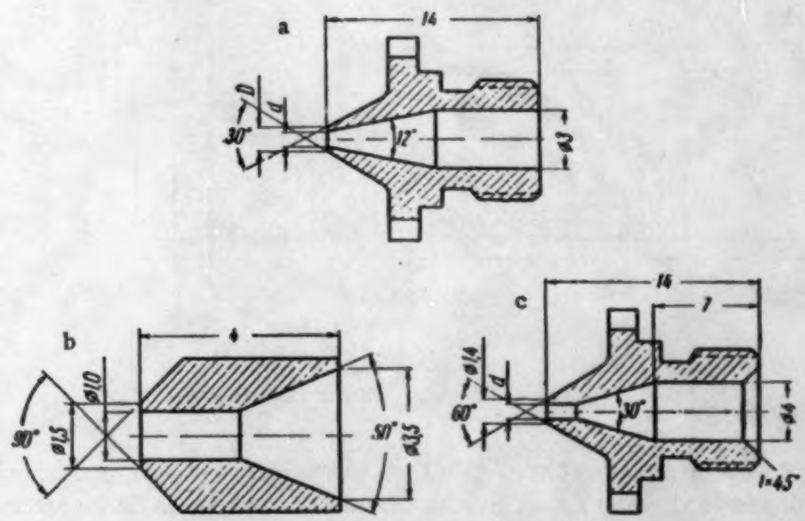

Fig. 1. Types of nozzles investigated.

The information to be found in the literature on the jet's power effect on the flapper is insufficient in many cases [2] and somewhat inaccurate, since it contains data for a limited range of the parameters which characterize the amplifier (valve diameter, working fluid pressure, gap between valve and flapper, etc.).

In this paper we present the results of the experimental investigation of jet power effect on nozzles, obtained with increased pressures of the working fluid, positive and negative working fluid temperatures and for a broad range of variation of the gaps between nozzle face and flapper.

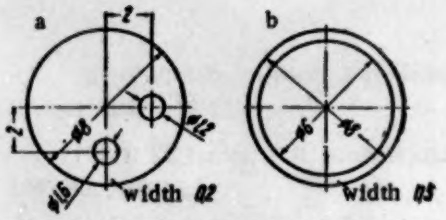

Fig. 2. Membrane a and collar b of the laminar valve.

The three types of nozzles shown in Fig. 1 were investigated.

The program for testing to determine the jet power effect on the flapper was divided into two series. In the first of these, testing was carried out with constant pressure drops on a valve with variable cross section, i.e., on a nozzle with a flapper. Nozzles of the first type (Fig. 1a) were tested, with d = 0.6, 0.8, and 1 mm and D = 0.95, 1.15, and 1.35 mm, respectively. The purpose of this series of tests was to determine the discharge Q of working fluid through the variable-cross-section valve and the force N on the flapper as functions of the gap h between the nozzle face and the flapper for various constant pressure drops Δp across the valve.

The second part of the testing was carried out for sets of nozzle-flappers with valves of constant cross section. For the latter, we set up a laminar valve (valve block) composed of a series of membranes a clamped between collars b (Fig. 2). The membranes were collected into blocks in which their apertures were diametrically opposed. The number of membranes was so chosen that there was a flow through the valve of 1425 cm^3/min of the working fluid at a temperature of from 30 to 35°C and a pressure drop of 25 kg/cm^2 across the valve.

The second part of the testing program was made for various constant pressures p_0 at the input of laminar valves for nozzles of the third type, with diameters of 0.8 and 1 mm. We determined by this the discharge of working fluid through variable valves, the force on the flapper and the pressure p_1 in the intervalve chamber as functions of the gap h for various pressures p_0 and for constant atmospheric pressure p_2.

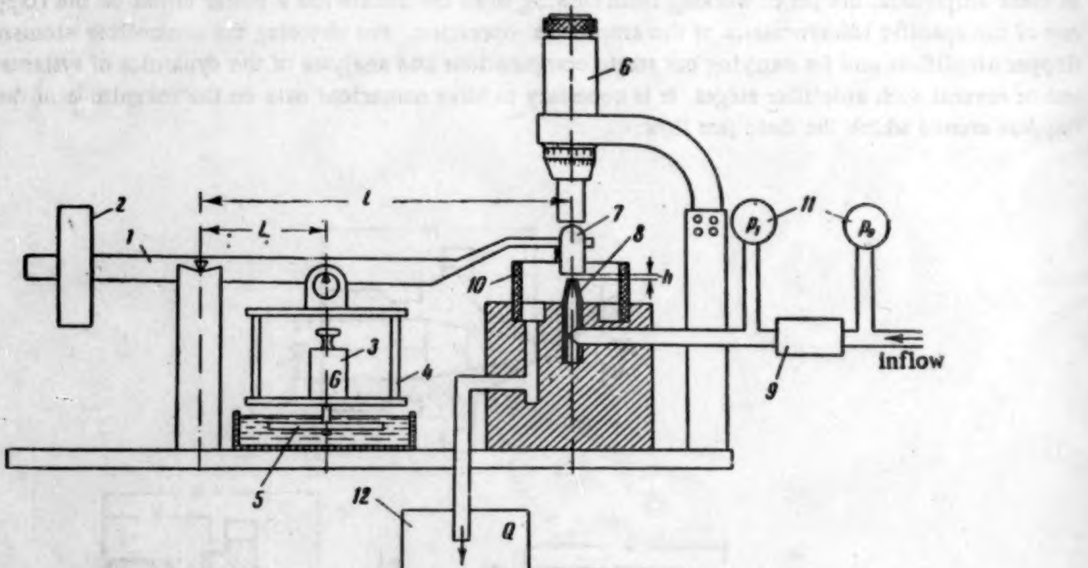

Fig. 3. Functional schematic of the experimental setup.

The working fluid for the tests was AMG-10 oil. In the first series of tests, the temperature of the working fluid was between 50 and 55°C, while three temperature intervals were chosen for the second series: −40 to −45°C, 45-50°C, and 80-90°C.

All the experiments were carried out on the setup whose functional schematic is shown on Fig. 3. This setup is analogous in many ways to that described in [2]. However, in contradistinction to the latter, the distance l_1 from the axis of rotation of arm 1 to the point of application of the load 3, which balances the force on flapper 7, is always maintained constant. Moreover, the setup used by us was supplied with damper 5, which was a disc with a 100 mm diameter immersed in oil. The basic arm (shaft) 1 and the weight pan 4 were suspended on knife-edge bearings.

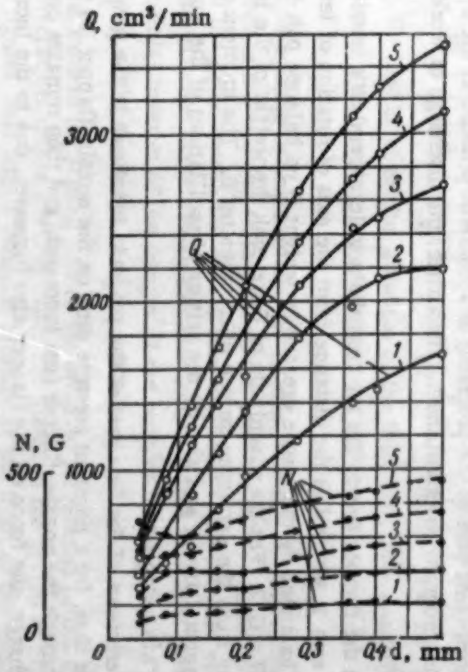

Fig. 4. Results of testing a nozzle of the first type with diameter of 0.8 mm with a working fluid temperature between 50 and 55°C: 1) for Δp = 10 kg/cm²; 2) for Δp = = 20 kg/cm²; 3) for Δp = 30 kg/cm²; 4) Δp = 40 kg/cm²; 5) for Δp = 50 kg/cm².

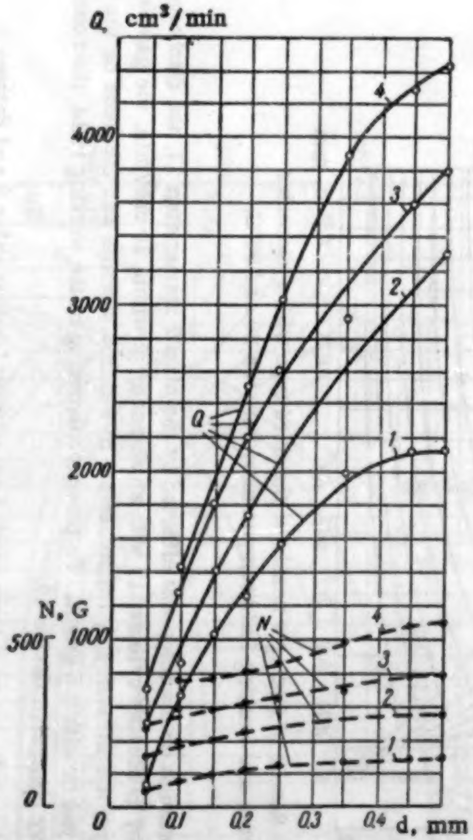

Fig. 5. Results of testing a nozzle of the first type with diameter of 1 mm for working fluid temperatures between 50 and 55°C: 1) for Δp = 10 kg/cm²; 2) for Δp = 20 kg/cm²; 3) for Δp = 30 kg/cm²; 4) for Δp = 40 kg/cm².

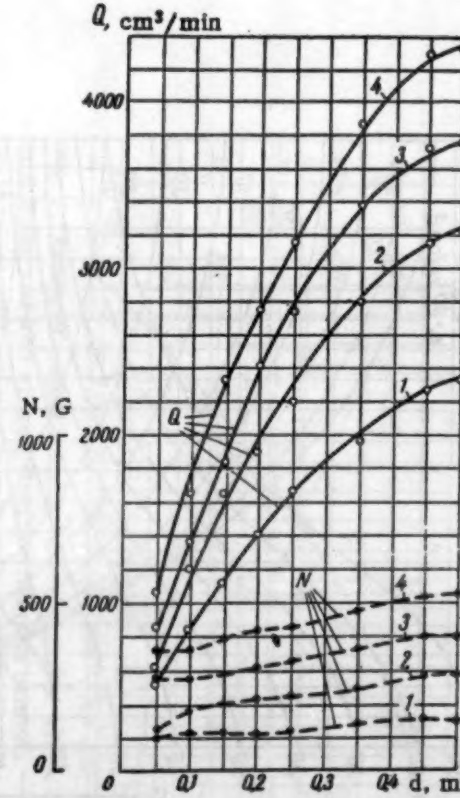

Fig. 6. Results of testing a nozzle of the second type with diameter of 1 mm for working fluid temperatures between 50 and 55°C: 1) for Δp = 10 kg/cm²; 2) for Δp = = 20 kg/cm²; 3) for Δp = 30 kg/cm²; 4) for Δp = 40 kg/cm².

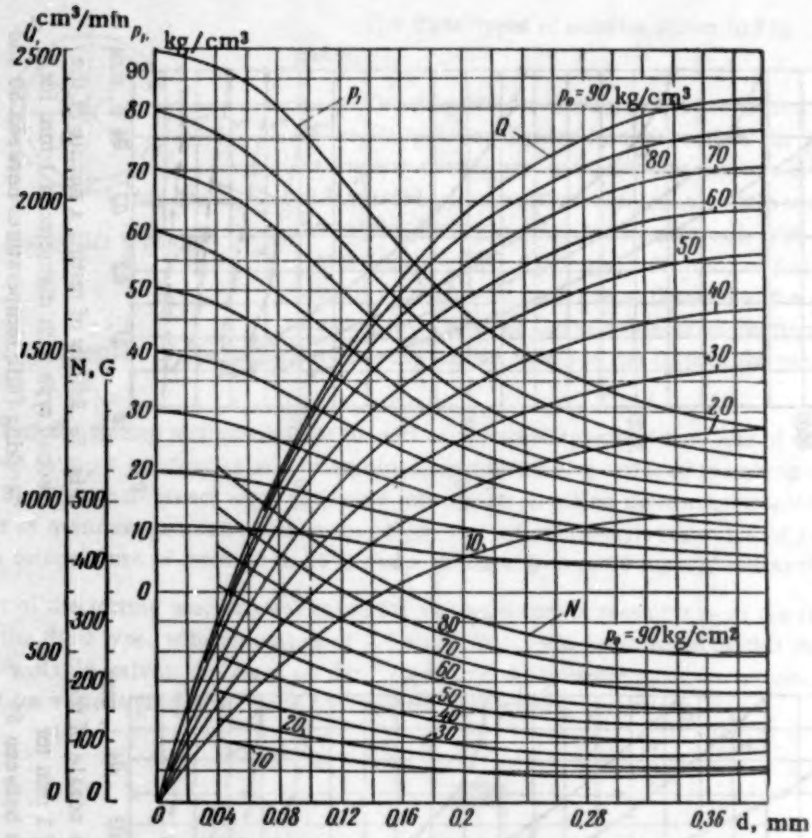

Fig. 7. Results of testing a nozzle of the third type with a diameter of 0.8 mm and working fluid temperatures between 80 and 90°C.

The measurement of pressure on the setup was effected by the class 2 manometers 11, the discharge of working fluid was measured by calibrated cylinder 12 with an accuracy of within 10 cm³/min, the gap between the nozzle face and the flapper was measured by micrometer 6 to within 0.005 mm and the force on the flapper was measured by the weight pan to within 5 grams. To prevent splattering of the working fluid, the nozzle and the flapper were surrounded by protective device 10.

The tests of the first part of the program were carried out without laminar valve 9 and damper 5.

The experimental method amounts to the following. With zero gap between the face of nozzle 8 and flapper 7, lever arm 1 is balanced by bob 2 in a horizontal position. Then, by means of micrometer 6, the given gap between nozzle and flapper is established and, by manometers 11, either the constant pressure drop, $\Delta p = p_1 - p_2$, on the nozzle-flapper (first part of the program) or a constant pressure at the input of laminar valve 9 (tests of the second part of the program) is established. Thereafter, the discharge of working fluid and the force on the flapper are measured.

The force N acting on the flapper due to the jet from the nozzle engenders a torque $M = Nl$ relative to the axis of rotation of lever arm 1, where l is the distance from the axis of rotation of lever arm 1 to the center of flapper 7. This torque is balanced by the torque created by weight G on balance pan 4, which is equal to $M_1 = Gl_1$. Since $l_1 = l/2$, then $N = G/2$. For the magnitude of G we took the weight of the load in balance pan 4 for which the flapper broke away from the detaining device of micrometer 6. The moment of break-away was fixed visually by the motion of flapper 7 itself, and also by the arising of oscillations of the working fluid pressure at the input to the nozzle.

The results of the experiments of the first part of the program are shown in the graphs of Figs. 4-6. The character of the curves shows that, for a constant pressure drop on the nozzle-flapper, as the gap h increases the discharge of working fluid through the nozzle-flapper first increases, and then remains constant. Simultaneously, with the increase of the discharge, the force on the flapper also increases, due to the increase in both the velocity

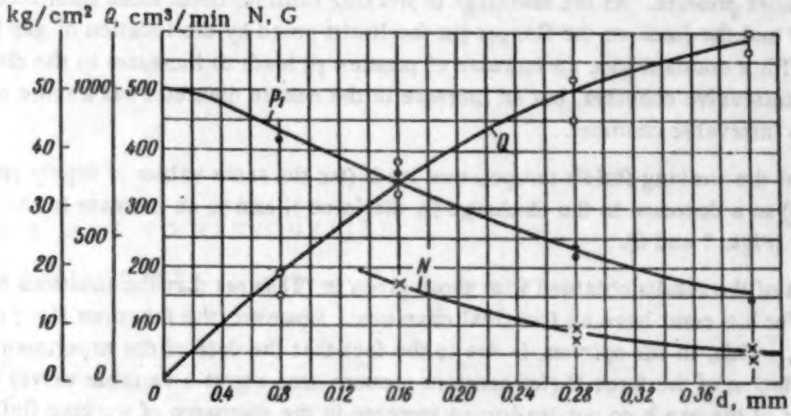

Fig. 8. Results of testing a nozzle of the third type with a diameter of 0.8 mm and with working fluid temperatures between −40 and −45°C.

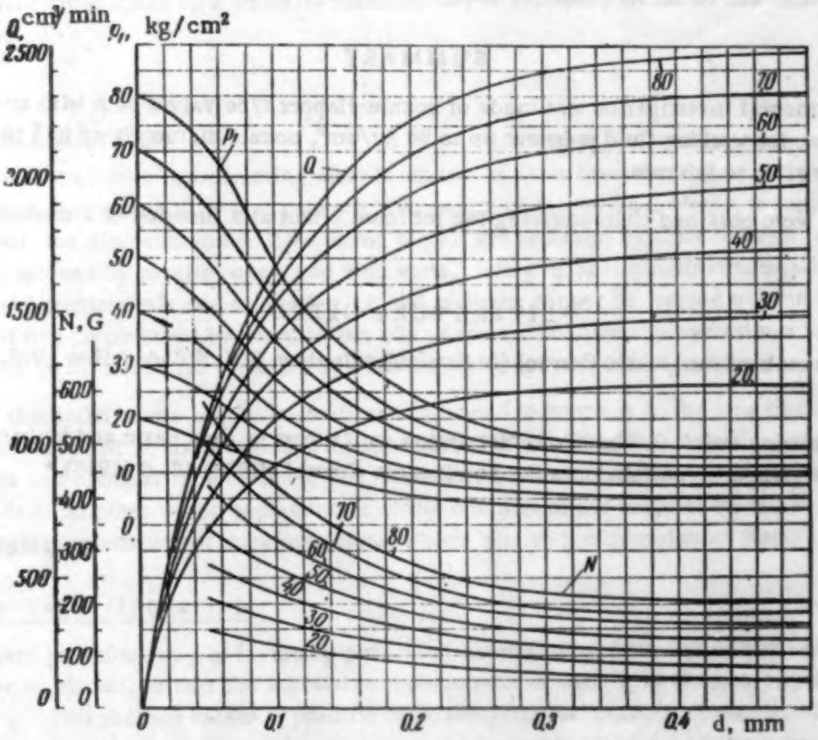

Fig. 9. Results of testing a nozzle of the third type with a diameter of 1 mm and with working fluid temperatures between 45 and 50°C.

and the mass of the fluid coming in contact with the flapper. On Fig. 4, for example, it is clear that when the increase in the discharge of the working fluid through the variable valve ceases, the increase of force on the flapper also ceases. If the gap between the nozzle face and the flapper remains constant, then an increase of the pressure drop on the variable valve and an increase in the nozzle diameter give rise to an increase of force on the flapper. The special features of the designs of the nozzles investigated did not show any noticeable influence on the magnitude of the discharge or of the force.

The results of the tests in the second part of the program for nozzles of the third type and with laminar valves, for various temperatures of the working fluid, are shown in the graphs of Figs. 7-9. It follows, from a consideration of these figures, that with a constant pressure at the input of the laminar valves there is, with an increase in the gap h, an increase in the discharge of working fluid but a drop in the force on the flapper and in

the intervalve chamber pressure. As the discharge of working fluid becomes more stabilized, the pressure in the intervalve chamber and the force on the flapper (in the limits posed by the location of gap h) approximate to constant values. With a constant gap, an increase of pressure p_0 leads to increases in the discharge, the force and the pressure in the intervalve chamber, but an increase in the nozzle diameter has a more telling effect on the pressure drop in the intervalve chamber.

A reduction of the working fluid's temperature leads (for the same values of supply pressure p_0, nozzle diameter d and gap h) to a decrease in the discharge Q, the force N and to an increase in the pressure p_1 in the intervalve chamber (Figs. 7 and 8).

A comparison of the results obtained with those given in [2] shows that the functions $N = f(h)$ for p_0 = const and also $N = f(p_1)$ for h = const have an identical character. However, the functions $N = f(h)$ for p_1 = const are essentially different. This, in our opinion, is due to the fact that the data of the experiments given in [2] comprise a range of variation of the force N (for constant pressure drop across a variable valve) within the limits of which the variations of the gap h do not lead to an increase in the discharge of working fluid through the nozzle-flapper.

The results obtained from our experimental investigation allow one to design hydraulic amplifiers of the nozzle-flapper type with increased working fluid pressures, larger nozzle diameters and larger gaps between nozzle and flapper, and so are useful for practical work.

SUMMARY

1. An experimental investigation was made of nozzle-flapper type valves both with and without constant cross-section valves, for working fluid pressures up to 90 kg/cm^2, nozzle diameters up to 1 mm and gaps between nozzle and flapper of up to 0.5 mm.

2. New data were obtained characterizing the jet force effect as a function of a number of system parameters.

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INVERSION AS A WAY TO RATIONALIZE CLASS H RELAY CIRCUITS

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Graphic inversion of plane circuits is carried out by the method of transfiguration, and the equality of this way with the inversion of spatial circuits is proved. The method cited opens up a formally reasoned way to transform plane circuits into spatial ones.

Posing of the Problem

The graphical method of transforming class H circuits to their inverses is the most widely used. The known method of inversion amounts to this, that each closed contour of the original circuit is replaced by a node. Simultaneously with this, the elements in the lines of the circuit are replaced by their inverses, and series connections of elements are replaced by parallel ones, and vice versa. Every class H circuit structurally consists of nodes and contours connected continuously one to another. Since contours cannot be formed without nodes, it is clear that an inverse circuit can be obtained by carrying out one of some characteristic operations: either all contours are replaced by nodes, or all nodes are replaced by contours, as is done in transfiguration.

In fact, if the nodes of the original circuit are expanded to contours in the new circuit, then the contours of the original circuit must be transformed to nodes since, first of all, in the new circuit as in the original one, nothing but nodes and contours is to be found and, secondly, in the transformation of the original circuit, only the nodes are expanded. In other words, replacement of the contours of the original circuit by nodes and replacement of nodes by contours are equivalent transformations, which lead to the inversion of plane circuits.

Inversion by Transfiguration

The standard graphical way of inverting consists of determining, from the original circuit, the locus of the nodes of the inverse circuit, so that the successive replacement of each loop (contour) by a node is carried out on the same drawing. This method makes it possible to orient graphically the dual circuit. In practice, it is more convenient to carry out the inversion by the successive replacement of each node by a contour, since the graphic construction in this case is carried out on a new drawing.

As an illustration, we carry out the inversion of the circuit, borrowed from [1] (Fig. 1), by individual steps from node to node in the original circuit M.

We first expand node 1. For the circuit of this node we determine that, in the first closed loop of the inverse circuit, there must be the following elements: $\bar{a}$, $\bar{c}$ and $\bar{f}$. We construct this loop in the form of a triangle (Fig. 2). From the circuit of node 2 we determine that, in the second loop, the elements $\bar{f}$, $e + \bar{d}$, and $\bar{k}$ must be contained.

We construct the second loop on the first so that element $\bar{f}$ is common to both loops. We carry out the expansion of the other nodes of the circuit in the same order. Naturally, the number of sides in a closed loop must equal the number of rays of the node in the original circuit.

Scheme M

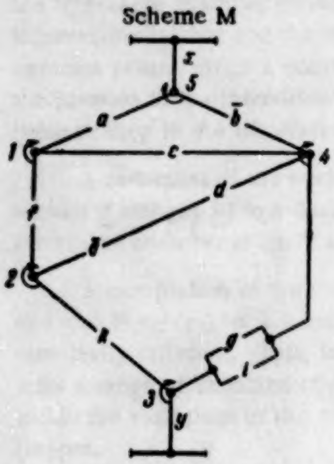


Fig. 1.

The expansion of each node should be carried out in one, previously established, direction (clockwise or counterclockwise) and the expansion should be begun each time with the element in common to the two neighboring nodes. If these requirements are met, the construction of the inverse circuit is carried out automatically. These requirements are consequences of the topological-geometric theory of circuits by which relay circuits are considered as subsets of the set of connected graphs [2].

The beginning and end of the circuit are determined in the following way.

The shortest paths from X to Y in the initial circuit were $af\bar{k}$ and $\bar{b}(g+i)$. Consequently, the beginning of the inverse circuit must be the point at which all the elements of the first short circuit are connected in parallel $(\bar{a} + \bar{f} + k)$, and the end must be the point at which all the elements of the second circuit are connected in parallel $(b + \overline{gi})$. Finally, the concepts of "beginning" and "end" of a circuit are purely conventional. These are the points at which the circuit is supplied.

We now place the circuit obtained over the original one (Fig. 3). Having convinced ourselves that the number of elements in both circuits is the same and that all elements are replaced by their inverses, we compare the structural admittances of the two circuits $(G_{x-y}; G_{v-w})$. If $G_{x-y} = 1$, then $G_{v-w} = 0$ and conversely. Consequently, the circuits are inverses, q.e.d.

Scheme N

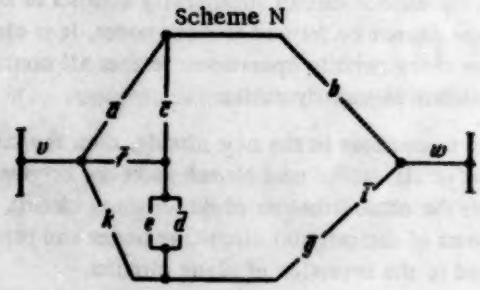


Fig. 2.

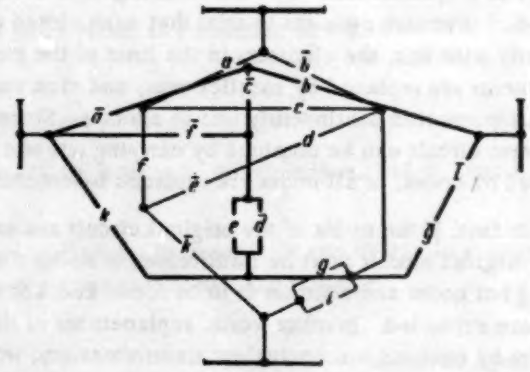


Fig. 3.

Special Features of Graphic Inversion

Obtaining inverse circuits in the class of the simplest pi-section circuits presents no difficulty, and is carried out either analytically or graphically with the same ease. For example, we represent graphically the circuit described by the equation $F = n(ab + cd)$ (Fig. 4a).

We now invert this circuit graphically by replacing the sole node in it by a loop, while simultaneously replacing all the elements making up the branches of the original node by their inverses (Fig. 4b).

We then write the equation satisfied by the inverse circuit thus obtained:

$$\bar{F} = n + (\bar{a} + \bar{b})(c + \bar{d}).$$

If we write the inverse equation

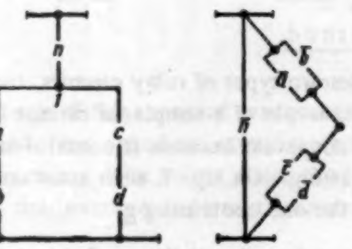
$$F = n(ab + cd),$$

we can convince ourselves that the expressions are isomorphic.

Two conclusions follow from this:

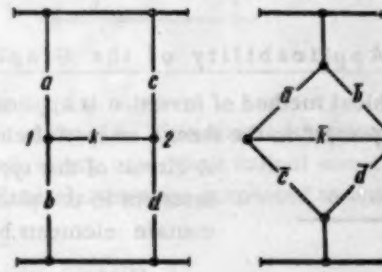
1) The inversion of the original circuit by graphic means was correct.

2) The search for the supply points of the inverse circuit obtained should be carried out by comparing both circuits and, if the current path in the normal circuit was <u>anb</u> then, in the inverse circuit, the supply point will be the point at which the inverses of these same elements are connected in parallel ($\bar{a}+\bar{n}+\bar{b}$), i.e., all the inverses of these elements are connected in parallel.



a b

Fig. 4.



a b

Fig. 5.

We now carry out the graphic inversion of the simplest H-circuit (Fig. 5). The analytic expression which describes the original circuit (Fig. 5a) has the form:

$$F = a(b + kd) + c(d + bk).$$

This form of the equation is not isomorphic to the circuit structurally, but is isomorphic to it dynamically, in the sense of the processes occurring in it, since the increase in the number of terms in the equation over the number of circuit elements is fictitious.

We obtain the inverse of this expression analytically:

$$\overline{F} = [\bar{a} + \bar{b}(\bar{k} + \bar{d})][\bar{c} + \bar{d}(\bar{b} + \bar{k})] = \bar{a}\bar{c} + \bar{a}\bar{d}(\bar{b} + \bar{k}) + \bar{c}\bar{b}(\bar{k} + \bar{d}) +$$
$$+ \bar{b}\bar{d}(\bar{k} + \bar{d})(\bar{b} + \bar{k}) = \bar{a}\bar{c} + \bar{a}\bar{b}\bar{d} + \bar{a}\bar{d}\bar{k} + \bar{c}\bar{b}\bar{k} + \bar{c}\bar{b}\bar{d} + \bar{b}\bar{d} =$$
$$= \bar{a}\bar{c} + \bar{a}\bar{d}\bar{k} + \bar{c}\bar{b}\bar{k} + \bar{b}\bar{d} = \bar{c}(\bar{a} + \bar{b}\bar{k}) + \bar{d}(\bar{b} + \bar{a}\bar{k}).$$

If we write the analytical expression for the graphically obtained inverse circuit, starting with elements $\bar{c}$ and $\bar{d}$, we can easily convince ourselves that the inversion was carried out properly.

This example allows us to draw the following conclusion.

When the nodes of an original circuit are expanded into loops of the inverse circuit, the ends of the branches should be considered either as the following nodes of the original circuit or as supply points of the circuit, depending on where the branch under consideration ends. Thus, in the first branch of the first node one element <u>a</u> (and not the two elements <u>ac</u>) is contained; in the second branch of this node there is again just one element <u>k</u> contained, etc.

We now carry out the inversion of a somewhat more complicated circuit (Fig. 6a), using the graphical method.

By placing the circuit obtained (Fig. 6b) on the original circuit in such wise that the nodes of the resulting circuit are inside the loops of the original circuit, we convince ourselves that the inversion was properly carried out. This example allows still another important conclusion to be drawn.

If to the poles of the circuit there are connected, not two, but three or more branches, then the poles of the new circuit should be sought in the boundary circuits of the original circuit which contain the least numbers of elements, i.e., in the shortest circuits.

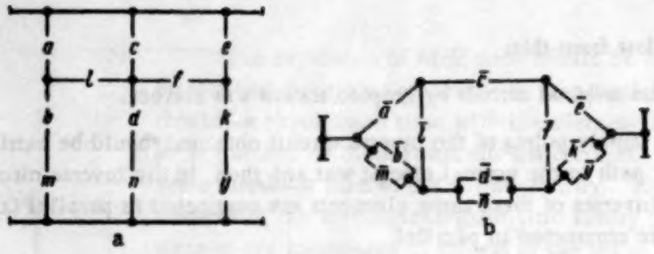


Fig. 6.

Domain of Applicability of the Graphical Method

The graphical method of inversion is applicable to all known types of relay circuits, including those which are called "nonplanar" in the theory, or spatial circuits. An example of a nonplanar circuit is given in Fig. 7.

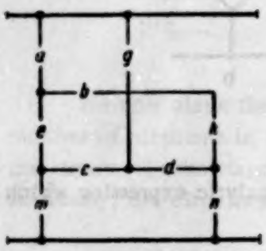


Fig. 7.

A circuit of this type is called nonplanar because it contains arms which do not intersect in the plane of the drawing. On Fig. 7, such arms are those which contain elements b and k and the one containing g.

However, it is known from relay circuit theory that any H-circuit can be expressed as the sum of several simple pi-circuits. This expansion is carried out by the method of determining the circuits which act on the given elements [3].

Thus, as applied to relay circuits, there are sufficient grounds for asserting that, in the set of these circuits, there cannot exist a circuit the dynamics of which would be impossible to represent without intersections in the plane of the drawing. In other words, one can assert, in correspondence with the method of determining the circuits which act on a given element, that circuits constructed for the same set of conditions and differing from each other by the presence in one of them of some greater number of single-valued elements, do not differ in theory, but only structurally, i.e., H-circuits are simplified sums of series-parallel circuits.

Inversion of Nonplanar Circuits

We now invert the nonplanar circuit of Fig. 7. A trivial way to invert such a circuit is to first decompose it and then to invert it by parts [4].

We decompose the given circuit into two circuits by determining the circuits on which element a and element g act (Fig. 8). Both circuits, the sum of which is equivalent to the original circuit, are laid on a plane without mutual intersections, and may be easily inverted, as is done in Fig. 9.

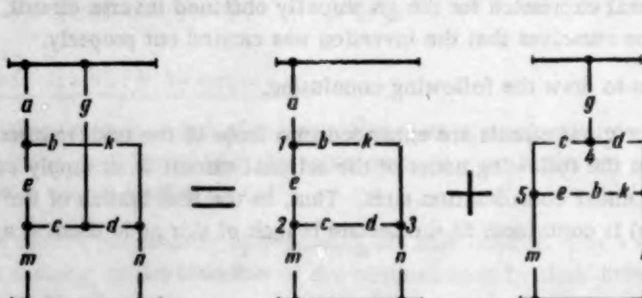


Fig. 8.

The inverted terms of the original circuit are multiplied, i.e., are joined in series. We then carry out a graphic simplification of the circuit, by which we obtain an inverse circuit with the same number of elements as in the original circuit. Since this cannot always be done we then, to convince ourselves that the circuit obtained is actually the inverse of the original one, carry out the operation of inversion on the inverted circuit and, since $f = F$, the result of this operation must be the original circuit.

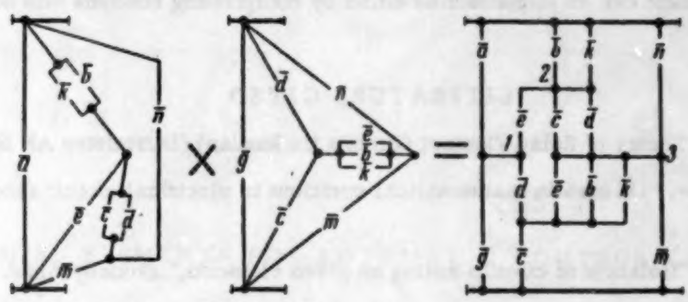


Fig. 9.

The direct result of this repeated inversion is the circuit of Fig. 10. This circuit is equivalent to the original one, since the closed loops ebkcd are completely identical and, in any case, their mutual connections can be considered as one contour. The points where the initial and final elements are connected to such a loop are also known. Certainly, the inversion is correctly executed.

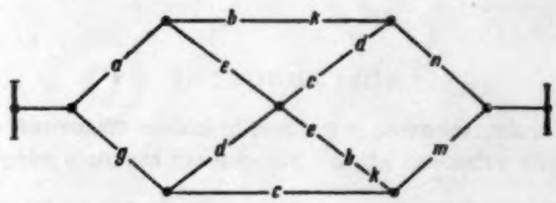


Fig. 10.

The circuit of Fig. 9, however, is inverse to the original circuit only by the existence of the transformation carried out on it. It is clear from what has been done that not every spatial circuit can have a dual, in the full sense of this definition.

SUMMARY

The present work provides a foundation for the following conclusions.

First, although the inversion of a nonplanar circuit by the application of our method did not lead to the obtaining of a dual, it is clear from the proof of inverseness given above that, by applying inversion as a way to synthesize sequential circuits, one can succeed in simplifying them until a spatial circuit is obtained.

Second, in spite of the fact that the method of transforming bridge circuits to series-parallel circuits by isolating the circuits which act on given elements makes it possible to describe any complex scheme by homomorphic analytical expressions without the use of special symbols (# and ⁞) and, seemingly, opens a path for applying, without limitations, the algebra of relay circuits to such expressions, the graphical method, in fact, turns out to be the most effective in practice. One can, with its help, obtain the desired results much more rapidly and with less expenditure of effort.

For example, the analytic expression which describes the circuit of Fig. 7 is

$$F = a\,[(e + bkcd)\,m + (bk + ecd)\,n] + g\,[(c + dkbe)\,m + (d + cebk)\,n].$$

The inversion of this expression is

$$\overline{F} = \{\overline{a} + [\overline{e}\,(\overline{b} + \overline{k} + \overline{d} + \overline{c}) + \overline{m}][(\overline{b} + \overline{k})(\overline{e} + \overline{c} + \overline{d}) + \overline{n}]\} \times$$
$$\times \{\overline{g} + [\overline{c}\,(\overline{d} + \overline{k} + \overline{b} + \overline{e}) + \overline{m}][\overline{d}\,(\overline{c} + \overline{e} + \overline{b} + \overline{k}) + \overline{n}]\}.$$

Simplifying this expression analytically is significantly more difficult than doing it graphically, as was done on Fig. 9.

Third, graphic inversion can be implemented either by compressing contours into nodes or by the transfiguration method.

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MAGNETIC LOGICAL ELEMENTS FOR AUTOMATIC CONTROL CIRCUITS

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The functional schematics of basic systems of logical elements based on magnetic cores and semiconducting diodes are considered.

Comparative evaluations of these systems are made.

INTRODUCTION

In many systems for the automatic control of productive processes, relays frequently play the part of the elements lying between the sensor elements (transducers) and the executive elements.

The basic purpose of the relays in such schemes is to execute various elementary operations which permit the requisite character of executive element control to be obtained as a function of the form of the signals applied to the relay inputs. The logical operations in this case as, to be sure, in the majority of discrete (digital) devices, are based on the binary code, for which only two possible output states are used. For a relay, these are the two different positions of the contacts.

Magnetic logical elements are advantageously employed to increase reliability, simplify usage and increase the length of service of such systems. Such elements have no moving contacts and are also characterized by two output states, namely, the presence or absence of an output voltage.

The basic simplest logical functions, on whose basis all possible control operations can, with sufficient flexibility, be implemented are the functions of "and," "or," "not," "memory," and time delay. We now briefly characterize these functions.

The "and" element has several input circuits, and a signal appears at its output only in case it has signals simultaneously at all its inputs. By its action, this element is analogous to a circuit consisting of several series-connected relay contacts. Such a circuit is closed only when all the contacts are closed.

The "or" element also has several input circuits, but a signal appears at its output whenever there is a signal applied to at least one of its inputs. This circuit is thus analogous to a circuit consisting of several relay contacts connected in parallel.

The "not" element has one input circuit, and a signal appears at its output only when there is no signal at its input and, conversely, where there is a signal at its input, its output signal disappears. This element is analogous to a relay with normally closed (break) contacts. The "memory"* element has two inputs and is characterized by the fact that, when a signal is applied to the first input, a signal appears at the output and remains there even after the signal at the input is discontinued. In order for the output signal to disappear, it is necessary that a signal, perhaps only a momentary one, be applied to the second input. Thus, a "memory" element is analogous to a polarized relay, or to a lock-up relay.

The time delay element can implement various functions, analogous to those characteristic of relays with

*This element can also be called a "dynamic memory" or a "dynamic trigger."

time delays. For example, a "switch-in delay" can be implemented, when the signal at the output appears with a definite lag after the appearance of an input signal, but disappears simultaneously with the removal of the input signal, or a "switch-out delay" can be implemented whereby the signal appears without any lag (i.e., at the output) but disappears with a definite lag.

The basic logical functions considered above can be implemented by the most diverse circuits and elements. With only diode circuits all the logical functions can be obtained, but the signals in such circuits will be damped, and intermediate amplifiers have to be connected in the circuits. The use of magnetic cores in combination with diode circuits allows one to obtain the logical functions together with the amplification necessary for operation of circuits with large numbers of individual elements. Here we shall consider only magnetic logical elements consisting of rectifiers and of cores with rectangular hysteresis loops.

Comparative Survey of Circuits of Magnetic Logical Elements

Circuits of magnetic logical elements, based on the use of cores with rectangular hysteresis loops, can be classified in various ways, depending on what is adopted as the basis for the classification. Thus, for example, as a function of the form of the output voltage, logical elements can be divided into pulse elements and elements with dc outputs. Another essential basis of classification by which different forms of logical element circuits can be grouped is the manner of connecting the load with respect to the elements' working windings. All circuits may be divided, on the basis of the load connection, into circuits with elements in series with loads (or circuits with voltage sources) and circuits with elements in parallel with loads (circuits with current sources).

To the class of circuits with elements in series with loads are the circuits of pulse elements which underlie the Ramey [1] fast-acting magnetic amplifier and the ordinary one- and two-core magnetic amplifiers in the amplifier and relay (switching) modes.

To the class of circuits with elements in parallel with loads are the pulse one-core transformer circuits, which are widely used in domestic computing and remote control devices [2], and circuits of pulse one-core elements with parallel connection of stages.

A. Pulse Element Circuits of Elements in Series with Loads

There are two types of logical elements whose operation is based on the Ramey circuit.

Figure 1 shows the functional schematic of the first of these, developed by the Westinghouse Corp.* in the United States [3]. With no signal at the input, the operation of this circuit is analogous to the operation of the Ramey circuit with shorted input. This is attained by having a flow of current in the circuits of diodes 1 and 2, given an auxiliary supply, the magnitude of these currents being somewhat greater than the magnetizing currents of windings w_1 and w_2, respectively. Thus, diodes 1 and 2 are open for the magnetization polarity-reversing cur-

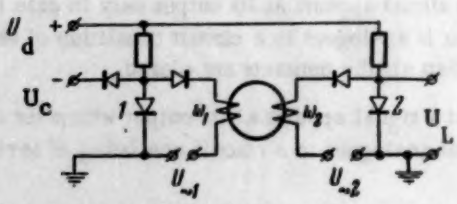


Fig. 1. Functional schematic of a repeater.

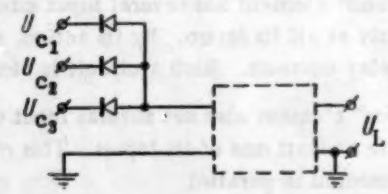


Fig. 2. Circuit for an "or" element.

rent of both windings. The switching in of the signal U_C, which is of sufficiently great magnitude, cuts off diode 1 and the diode circuit of the control winding, and output voltage U_L appears at the circuit's output, since the core will still not have its magnetic polarity reversed. Thus, the basic cell of elements of this type is a repeater with a half-period shift. Such a cell can form the basis of shift registers and pulse counters.

The "or" function is obtained very simply with elements of this type by connecting the signals through an auxiliary circuit which contains a number of diodes equal to the number of inputs (Fig. 2).

*This same principle of operation underlies the elements of the UNIVAC AF/CRC computer [4].

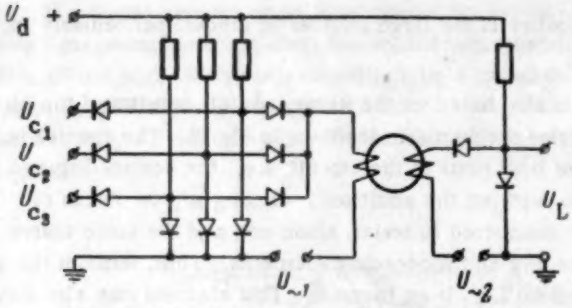


Fig. 3. Circuit for an "and" element.

The "and" function is obtained by means of the circuit shown in Fig. 3. In theory, the circuit permits the reliable logical multiplication of any number of inputs, but, with this, the input circuits must contain three times as many diodes as there are inputs.

The "not" element, or inverter, is obtained by means of a simplification of the circuit of Fig. 1. The circuit for such an element is shown in Fig. 4, where the control voltage U_c is connected in the control circuit instead of the supply voltage. For the crea-

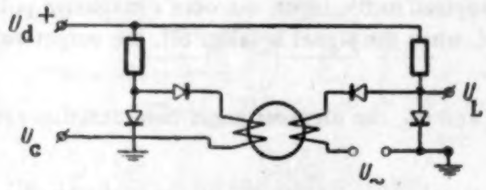


Fig. 4. Circuit for a "not" element.

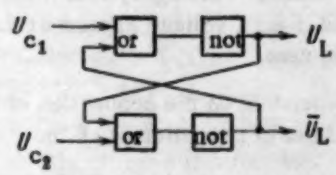


Fig. 5. Circuit for a memory element.

tion of a memory element in this system of logical elements, at least two "not" elements and two two-input "or" elements are necessary. The memory element has the form of a trigger (flip-flop), whose block schematic is given in Fig. 5.

The advantages of the logical elements just described are, first, the independence of reliable system operation on fluctuations of the voltage supply within quite wide limits (±30%), which is attained by virtue of the bias of the diode circuits and, second, the capability of obtaining the basic logical functions "and," "or," and "not" with a theoretically unlimited number of inputs on one and the same repeater cell without controlling or varying the parameters. This latter capability makes systems of these elements enormously flexible, and decreases the total number of elements necessary for implementing any given control problem.

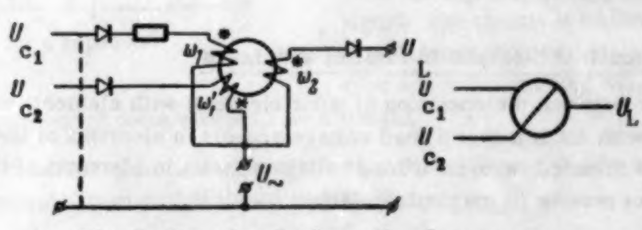


Fig. 6. Functional schematic of a repeater.

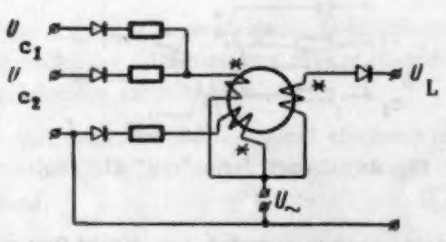


Fig. 7. Circuit for an "and" element.

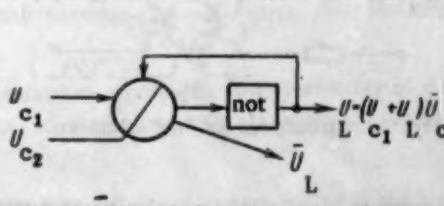


Fig. 8. Circuit for a memory element.

The disadvantage of this type of logical element circuitry is the large number of diodes, particularly in the logical elements which implement the "and" function.

The second type of logical element circuitry which is also based on the Ramey circuit consists of the circuits developed by the French company SEA [5]. The functional schematic is shown in Fig. 6. The special feature of this type of circuit is the common voltage supply for both parts of the circuit, i.e., the controlling and the operating parts. Due to this peculiarity it is impossible, without the additional winding w'_1, to obtain the logical function of signal repetition when the elements are connected in series, since one and the same source voltage is the magnetic polarity-reversing voltage for preceding and succeeding elements. Thus, without the additional winding, each element implements the "not" function, i.e., is an inverter. This element can also serve as the basis for shift registers and pulse counters, but with inversion from element to element. To obtain the function of repetition, one must connect winding w_1 via input 1 to the voltage source (the dashed line on Fig. 6) and, as the input winding, use winding w'_1. Then the ampere-turns of the signal will compensate the demagnetizing ampere-turns of winding w_1 and, when a signal is applied to the input, the core's magnetic polarity will not be reversed, i.e., a voltage appears at the output and, when the signal is taken off, the output voltage becomes equal to zero.

Thus, depending on the connection of windings w_1 and w'_1, the element under consideration can implement the function either of repetition or of "not."

Joining this element with an "or" circuit (Fig. 2) allows one to obtain the direct or the inverse (depending on the connections) "or" function for a theoretically unlimited number of inputs, just as with elements of the first type. The "and" function cannot be obtained directly with just one such element.

Figure 7 gives a circuit which allows the inverse "and" function to be obtained for two inputs, U_{c1} and U_{c2}. Thus, to obtain the direct "and" function of just two signals, it is necessary to connect one "not" element to the output of such an element. Consequently, in circuits requiring a large number of multi-input "and" elements, the use of logical elements of this type is less efficient than the use of elements of the first type, since a much larger number of elements is required here. For constructing a memory element in this system of logical elements, at least two standard elements are also necessary (Fig. 8).

Logical elements of this type permit a voltage source fluctuation of ±10%. The advantage of this type of element as compared with the first type is the decrease in the number of diodes. However, the logical elements of the second type also have disadvantages, namely: they are very sensitive to source voltage fluctuations and have a less flexible structure, i.e., a large number of elements is required when the use of "and" elements with several inputs simplifies the circuit as a whole.

B. Pulse Element Circuits of Elements in Parallel with Loads

The basic difference between the operation of pulse elements with elements in parallel with loads from that of elements in series with loads is that a load voltage appears in elements of the parallel type only when the core's magnetic polarity is reversed, whereas a load voltage appears in elements of the series type only when the core is saturated and cannot reverse its magnetic polarity.

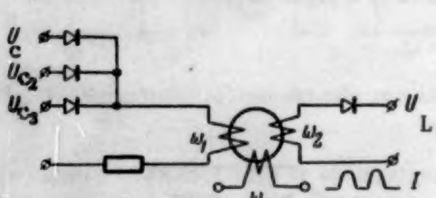


Fig. 9. Circuit for an "or" element.

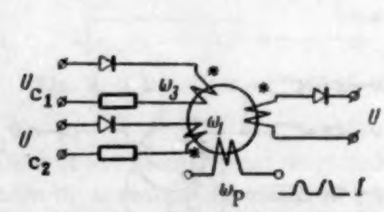


Fig. 10. Circuit for a "not" element.

In the group of circuits of logical elements with loads connected in parallel, one should first mention the transformer element [6], which is widely used in computing technology and is being introduced into remote control technology. Figure 9 shows the functional schematic of such an element, which is implementing the "or" function with three inputs. Single-polarity pulses from a current source pass through the core via special winding

w_p. If a signal appears on at least one of the inputs $U_{c1}-U_{c3}$ in an interval between these pulses and reverses the core's magnetic polarity, then the succeeding pulse from the source winding reestablishes the previous saturated state of the core, wherein load voltage U_L appears on load winding w_2 at the moment of magnetic polarity reversal by the supply pulse. With the core's magnetic polarity reversed by the input signal, the voltage in the output winding is blocked by the diode. Using elements of this type, one can construct shift registers, pulse counters and other circuits.

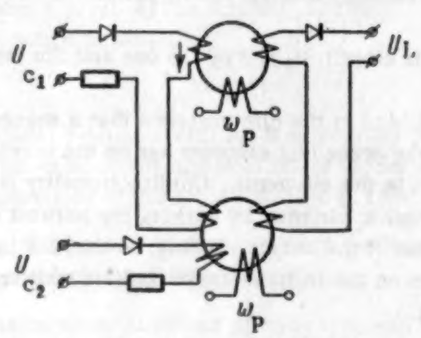


Fig. 11. Circuit for an "and" element.

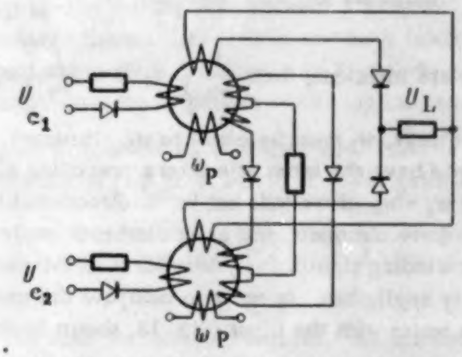


Fig. 12. Circuit for a memory element.

To obtain the "not" function in an element similar to the one just described, special winding w_3 (Fig. 10) is introduced, the ampere-turns of this winding being directed toward the ampere-turns of winding w_1, blocking its action when a signal is applied to the input of winding w_3. Thus, signals U_{c1} and U_{c2} must be introduced in order to obtain the "not" function in this element.

The element which implements the "and" operation in this system of logical elements has two cores (Fig. 11). The element of the circuit of Fig. 11 has a nonzero output voltage only when the first core only has its magnetic polarity reversed. This occurs only when there are signals on both inputs, U_{c1} and U_{c2}, which induce, in the second core, ampere-turns which are equal and oppositely directed.

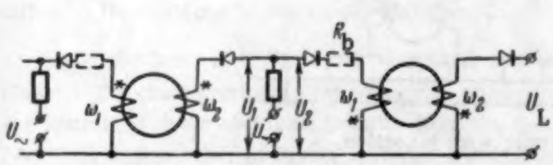


Fig. 13. Circuit for a repeater.

For the creation of an "and" element which permits the logical multiplication of a large number of signals, the circuit is further complicated. Because of this, circuits with logical elements of the transformer type are so constructed that the use of "and" elements is avoided, or so that the number of these elements be minimal.

The memory element in this system of elements also requires the use of two cores, as do all the other circuits of pulse logical elements. Its circuit is shown in Fig. 12.

The basic advantage of logical elements of the transformer type is that they operate reliably with fluctuations of the supply voltage within wide limits (±50%). This is explained by the fact that the integral obtained at the output of each element, is determined only by the magnitude of $2w_2\Phi_s$ of the core and, starting with some necessary minimum quantity of magnetic polarity-reversing ampere-turns, does not depend on the magnitude of these ampere-turns.

The disadvantage of the logical elements of the transformer type is the low flexibility of the circuits when the "and" element is not used or the increase in the number of elements when elements implementing this function are used.

We turn briefly to the elements where the load is connected directly in parallel with the working winding. The circuit of two such elements, one of which is connected to the output of the other, is shown in Fig. 13, This circuit can form the basis for shift registers or pulse counters. It possesses the following special features.

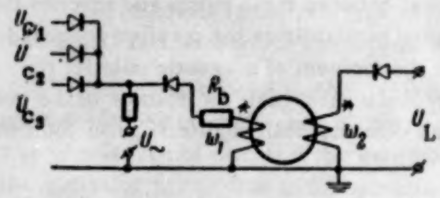


Fig. 14. Circuit for an "and" element.

The circuit of one such element is a repeater, in which an output voltage appears when its core has its magnetic polarity reversed in a working half-period, since the load is connected in parallel with the working winding. When another such element is connected in as the load, the working half-period of the controlling element is the controlling half-period of the controlled element. In this half-period, the reversal of magnetic polarity of the cores of the first and second elements occurs simultaneously and, if the elements are identical, then $\frac{1}{w_1}\int_0^{t_1} u_1\,dt = \frac{1}{w_2}\int_0^{t_2} u_2 dt$. In this circuit, u_1 and u_2 are one and the same voltage and, seemingly, w_1 must be equal to w_2. However, this would lead to the circumstance that a succeeding element would have the same effect on a preceding element as the preceding element has on the succeeding one. Thus, for $w_1 = w_2$, there will not be unidirectionality of action in the elements. Unidirectionality is easily obtained in these elements, just as in elements implemented by other circuits, by making the number of turns of the input winding significantly smaller than the number of turns of the output winding, so that the back effect is effectively negligible. In order to dissipate the excess voltage on the input element, ballast resistor R_b is connected in series with the input (Fig. 13, shown in dotted lines).

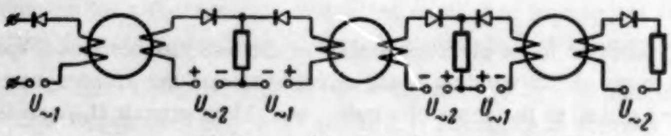


Fig. 15. Circuit for a shift register without inversion.

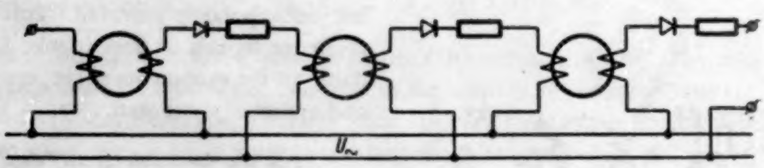


Fig. 16. Circuit for a shift register with inversion.

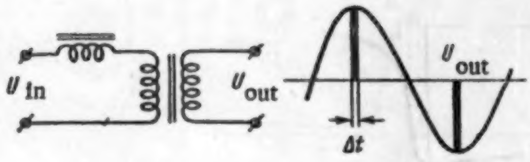
It must be mentioned that the circuit of Fig. 13, which is being considered here as a circuit with the load connected in parallel (a circuit with a current source), can also be considered as a circuit with the load connected in series (circuit with a voltage source) if, as the input and output quantities, we take the voltages on the impedances* and not on the choke windings. Depending on what one considers as the input and output quantities (voltages on the impedances or on the chokes), the functions "and" and "or," implemented by this circuit, are interchanged. Thus, if this circuit is considered as a parallel one, then the circuit with the diodes connected to produce the "and" function for any number of inputs will have the same form as the circuit with the diodes connected so as to give the "or" function with a series-connected load. Figure 14 shows such a circuit with three inputs. With a parallel connection of the load, the "or" function is obtained in a more complicated way, similar to the circuit which implements the "and" function using elements with series-connected loads (Fig. 3).

In carrying out our purpose of making a comparative survey of the pulse magnetic logical element circuits, we formulate the following conclusions.

1. In all the circuits considered for creating memory elements, at least two simple elements, forming the arms of a flip-flop, are required, and none of the circuits has any essential superiority over the others.

2. The two-winding logical elements with biased diode circuits (the circuits of Figs. 1-5) have single-core modifications of the basic logical functions "and," "or," and "not" for any number of inputs, and operate reliably within a broad range of supply voltage fluctuation (±30%), but have a larger number of diodes than the other circuits.

*It is easily shown that the circuit of Fig. 1 can be taken to this same form.



a b

Fig. 17. Pulse supply: a) The circuit; b) The form of the output pulses.

3. The three-winding logical elements (the circuits of Figs. 6-8) have fewer diodes, but permit fluctuations of only ±10% in the supplied voltage, and do not have single-core modifications of the logical function "and" for an arbitrary number of inputs.

4. The transformer logical elements (the circuits of Figs. 9-12) operate reliably over a wide range of supply voltage fluctuations (±50%) and have fewer diodes (in comparison with the circuits of Figs. 1-5), but have no single-core modifications of the logical "and" function.

5. Logical elements with loads in parallel permit the easy implementation of the "and" and "or" functions with any number of inputs and with fewer diodes than the elements of Figs. 1-5, but have lower gains and permit smaller fluctuations of the supply voltage (±10%).

6. All the types of logical elements considered are roughly equivalent from the point of view of power delivered for equal physical dimensions.

7. The repeater elements of each type can be used as the basis for shift registers (cf. for example, Figs. 13, 15 and 16). To convert them to ring counters, it is necessary to supply counting pulses, rather than supply pulses, to the circuit elements and, at their inputs, to supply starting pulses. Registers constructed of elements of the first, third, and fourth types shift pulses without inverting them, while elements of the second type invert the shifted pulses.

8. The dimensions of all the types of logical elements considered can be significantly lessened if short pulses of the same frequency are supplied to them (Fig. 17b). It is necessary to mention, at the same time, that the pulse supply (Fig. 17a) also executes simultaneously, the function of a stabilizer, since it gives a constant magnitude of $\int_0^t u_L\, dt$. This is particularly important for circuits of the second type, which are relatively sensitive to fluctuations in the supply voltage.

On the basis of all that has been said, one may conclude that, of all the systems of logical elements considered, the most promising for industrial application is that whose elements have biased diode circuits, since the system of these elements is quite flexible, i.e., allows one to create any modification of "and," "or," and "not" for any number of inputs and with a minimum number of cores, and permits large fluctuations in voltage supply. The disadvantage of this system is the large number of diodes which, when silicon diodes are used for objects working in difficult temperature conditions, raises the cost of these elements.

In industrial objects, where the dimensions of the elements are of significance, it makes sense to use systems of small-size elements, drawing from a pulse source.

C. Circuits of Logical Elements with DC Outputs

Single-core circuits in which the feedback coefficient is brought to the necessary value by means of a condenser are used as logical elements by the General Electric Company [3]. Figure 18 shows the circuit of such an element. Thanks to the presence of the condenser, the average value of the current in the feedback winding is much higher than in the working winding.

By varying the number of turns in the feedback winding and the capacity of the condenser, one can change the feedback gain of this circuit and obtain different slopes of the element's input-output characteristic. With a sufficiently high feedback gain, the element enters the relay (switching) mode, and can be used as a memory element. This same circuit can be easily made the base of the logical "not" element (the characteristic of Fig. 19) and an "or" element. The "and" element, in this system of logical elements also, is more complicated than the other elements. For reliable operation of the "and" element, it must have as many cores as there are inputs (Fig. 20), so that the full output voltage only appears when all the cores have their magnetic polarity reversed.

A disadvantage common to all these circuits is the necessity of using condensers, which either significantly increase the size of the elements or are the cause of variations in circuit parameters, which can lead to the

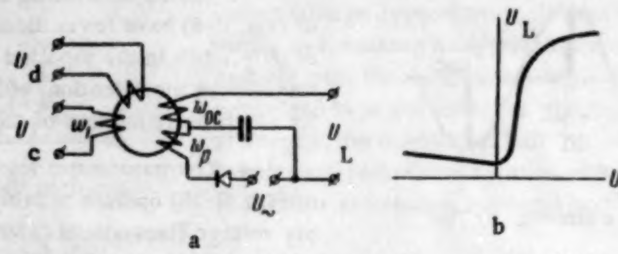


Fig. 18. a) Functional schematic of a repeater; b) repeater characteristics.

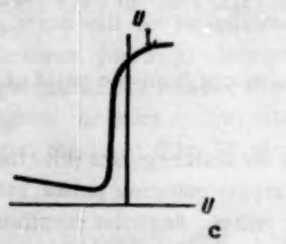


Fig. 19. Characteristic of the "not" element.

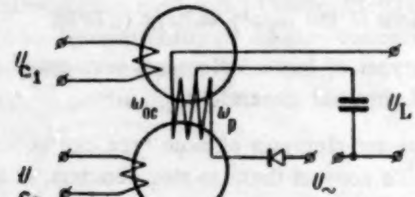


Fig. 20. Circuit for an "and" element.

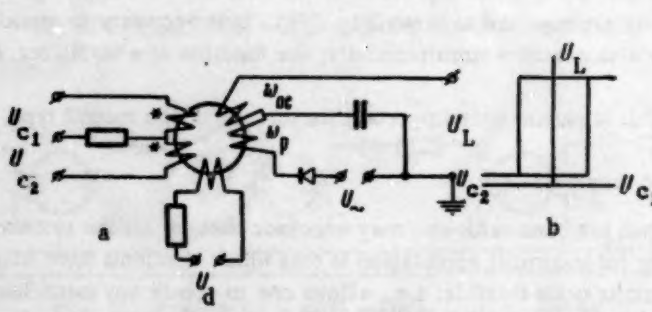


Fig. 21. a) Circuit of a memory element; b) characteristic of the memory element.

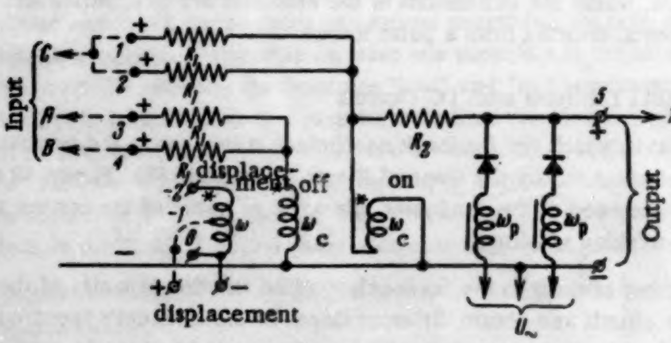


Fig. 22. Circuit for a universal element.

necessity of controlling and adjusting the circuits during the period of usage. The "and" element, moreover, is relatively complex to use, since it contains several cores with individual and common windings. In our opinion, the memory element is the only one clearly superior to the pulse elements considered above, since only in this system of logical elements can it be implemented in a one-core modification.

The memory element (circuit and characteristic in Fig. 21a and b) operates reliably for significant fluctua-

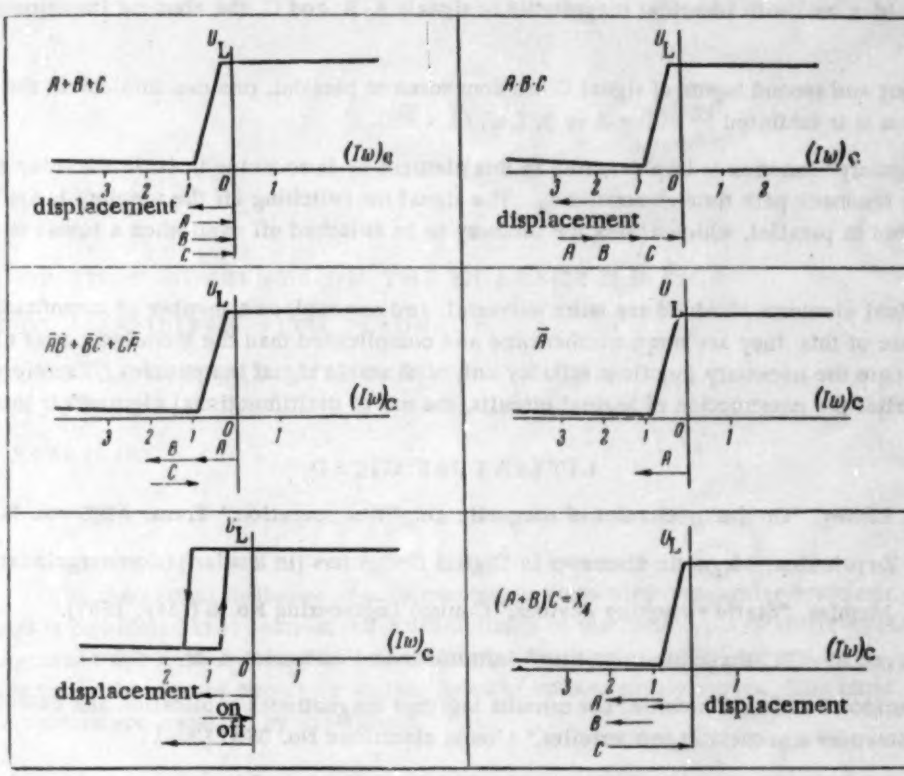


Fig. 23. Characteristics of the universal element.

tions in supply voltage, with the bias for the windings coming from a common source. It has two input windings, one for switching on, the other for switching off.

In our opinion, it is sometimes advantageous to use this modification of the memory element in conjunction with pulse "and," "or" and "not" elements.

D. Logical Elements Based on Ordinary Magnetic Amplifiers

The General Electric Company [3], in addition to single-core elements which execute simple logical functions, also developed logical elements which implement more complex functions. These elements are ordinary magnetic amplifiers with several control windings and combinations of input impedances. One of the control windings is used for biasing.

The circuit of a universal element, connected for the implementation of several logical functions, is shown on Fig. 22. This element has three inputs A, B, and C, and one output. Inputs A and B are applied in parallel to one of the two control windings. Input C is supplied to the second control winding, which can be connected in opposition relative to the first.

The two impedances R_1 in the circuit of signal C allow, by switching of these resistors, the magnitude of the controlling ampere-turns of signal C to be varied by a factor of two for one and the same signal level.

The circuit also contains a step-by-step control of the magnitude of the bias, by means of which the ampere-turns of the bias can be varied from zero to three times the unit signal ampere-turns.

Figure 23 gives the element's input-output characteristics for some of the logical functions which can be implemented by this element. The "or" function for three signals, A + B + C, is implemented by applying signals to one of the C inputs and to the A and B inputs. The ampere-turns of the bias must equal the ampere-turns of any of the three signals.

To obtain the logical "and" function, A · B · C, the magnitude of the bias ampere-turns must be triple that of each of the signals. The function of inversion (negation), "not," is obtained by applying a signal to input A or input B and with no bias.

With no bias, and with identical magnitudes of signals A, B, and C, the element implements the function AB + AC + BC.

If the first and second inputs of signal C are connected in parallel, one can implement the function of transmitting C unless it is inhibited by either A or B, i.e., $(\overline{A} + \overline{B})C$.

The "memory" function is implemented in this element by introducing in it, in the relay mode, an additional positive feedback path through resistor R_2. The signal for switching off the memory is applied to inputs 3 and 4, connected in parallel, which allows the memory to be switched off even when a signal to switch on is present.

The logical elements obtained are quite universal, and can replace a number of monofunctional elements but, just because of this, they are more cumbersome and complicated than the monofunctional elements. Moreover, they execute the necessary functions reliably only with stable signal magnitudes. Therefore, if the plug-in principle underlies the construction of logical circuits, the use of multifunctional elements is inefficient.

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INFLUENCE OF EDDY CURRENTS ON THE CHARACTERISTICS OF MAGNETIC AMPLIFIERS WITH FEEDBACK AND LOW CONTROL CIRCUIT IMPEDANCE

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The theoretical influence of eddy currents in cores with rectangular hysteresis loops is considered as it bears on the characteristics of the basic types of single-cycle magnetic amplifiers with feedback, including fast-acting amplifiers, for low impedance in the control circuit for various forms of voltage supply curves. The basic conclusions are borne out by experiment.

Magnetic alloys with rectangular hysteresis loops (50NP, 65NP, and others) are widely used in contemporary magnetic amplifiers. This allows a significant decrease in the weight and dimensions of an amplifier for a given power, a lowering of its inertia and an improvement in the linearity of its input-output characteristic. The characteristics of such amplifiers, particularly when high-quality germanium and silicon diodes are used, depend essentially on the form of the portions of the dynamic hysteresis loop by which the magnetic state of the amplifier's cores varies [1]. Deviations of the form of the parts of the dynamic hysteresis loop from the form of the corresponding static hysteresis loop can be due to eddy currents, magnetic viscosity and inhomogeneity of the magnetic properties of the core material along its cross section.

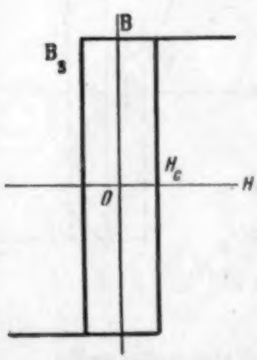

Fig. 1.

The present work is devoted to the theoretical consideration of the effect of one of these factors, namely, eddy currents, on the characteristics of the basic types of magnetic amplifiers with feedback, in the case when cores with ideal rectangular static hysteresis loops, such as shown on Fig. 1, are used. It is assumed that the ratio of the outer to the inner diameter of the cores is sufficiently small so that one can ignore the inhomogeneities of their magnetization due to the decrease in magnetic field strength as the point of the core under consideration is removed further from the core's geometric center [2].

The mode of operation of a magnetic amplifier and the effect of eddy currents depend essentially on the impedance of the control circuit. In this work, we consider the basic two-half-period magnetic amplifiers (Fig. 2) and also fast-acting magnetic amplifiers following the circuit of Fig. 3, all with very small impedances R_c in the control circuit:

$$R_c < \frac{w_c^2}{w_\sim^2} R,$$

where $w_\sim$ and w_c are the number of turns in the working winding and the control winding, respectively, and R is the active impedance of the working circuit, equal to the sum of the load impedance R_L, the forward impedance of the diodes R_d and the working winding impedance R_0. With this assumption, R_c cannot limit the magnitude of the current at double frequency directed into the control circuit. We consider load impedance R_L to be purely active, forward impedance R_d of the diodes to be constant and the back impedances to be infinitely large.

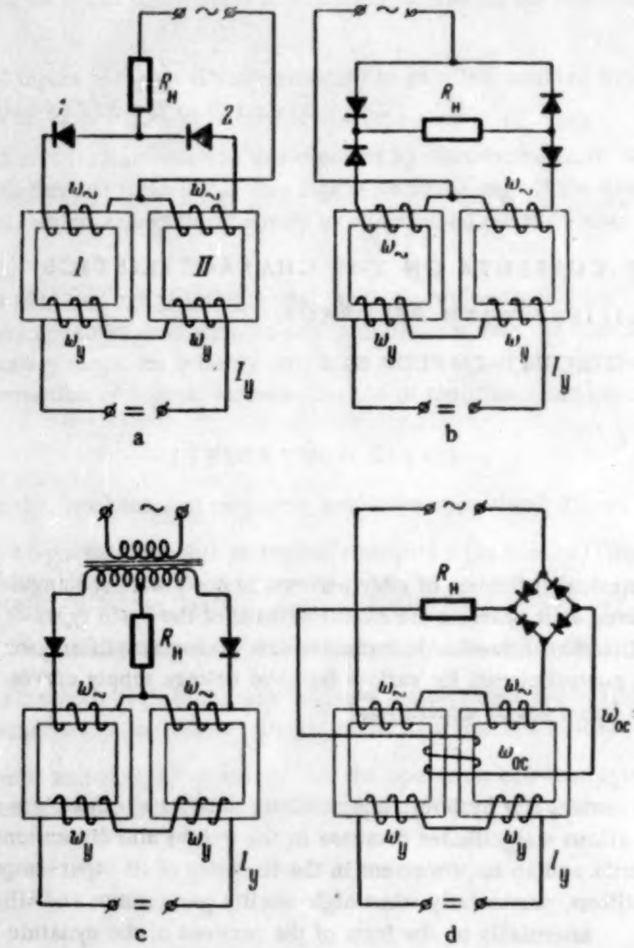

Fig. 2.

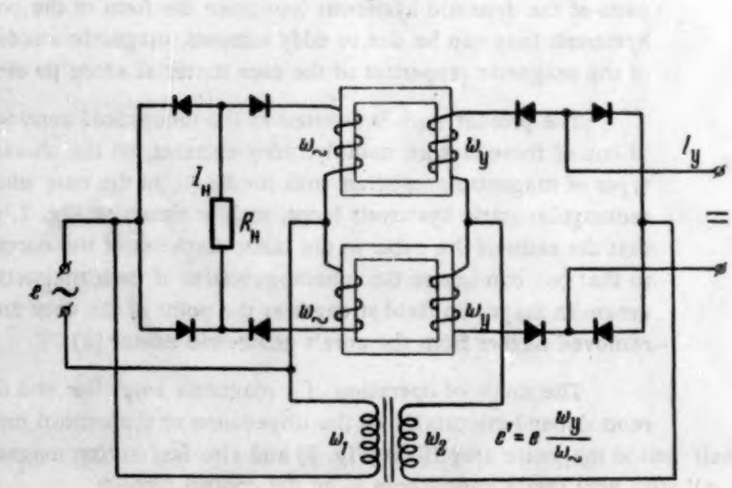

Fig. 3.

As the basis of our accounting for the effect of eddy currents, we take the equations for the dynamic hysteresis loop which were obtained by the author in [3] for various modes of magnetic polarity reversal of cores with rectangular static hysteresis loops. From Eqs. (2) and (3) of [3] we can obtain, in general form, the following expression for the dynamic hysteresis loop:

$$H = \pm H_0 + 5\pi\gamma d^2 \left(1 \pm \frac{B}{B_s}\right) \frac{dB}{dt} 10^{-10}, \tag{1}$$

where H_c is the static coercive force, B_s is the saturation induction, γ is the specific electrical conductivity, and d is the material's thickness.

The plus signs in formula (1) correspond to increasing induction, i.e., to the ascending arm of the hysteresis loop, and the minus signs correspond to decreasing induction, or to the descending arm of the hysteresis loop.

Double-Half-Wave Amplifiers with Sinusoidal Voltage Supply

With the assumptions made above, all the circuits of Fig. 2 have identical characteristics [1,4] and so we may limit our consideration to the first of them (Fig. 2a).

We give the subscripts 1 and 2 to the parameters appertaining to cores 1 and 2, respectively. We now consider the half-period when the supply voltage $e = E_m \sin \omega t$ through diode 1 is applied to winding $w_\sim$ of the first core (Fig. 2a). Under the action of this voltage, induction B_1 varies from some initial value B_{10} for $\omega t = 0$ to the saturation induction B_s for $\omega t = \alpha$, after which load current i increases by a jump from the low value of magnetizing current i_μ to a value determined by the active impedance R of the load circuit. Simultaneously with the variation in B_1 there is induced, in winding w_c of the first core, an emf e_2 which is applied to winding w_c of the second core and forces its induction B_2 to vary from the initial value B_s to value B_{20} at the end of the half-period. In the following half-period, the two cores interchange roles. In the steady state, $B_{10} = B_{20}$.

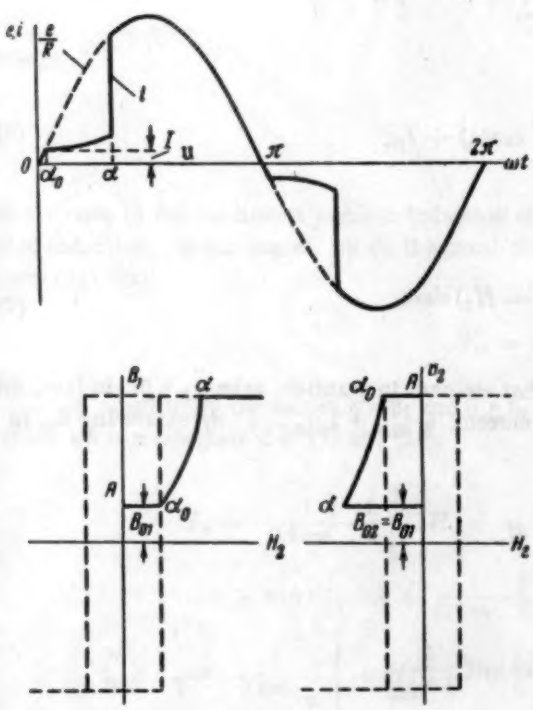

Fig. 4.

The character of change of load current i and the magnetic state of the cores during one half-period, with the effect of eddy currents taken into account, is shown in Fig. 4.

Field strengths H_1 and H_2 are induced, respectively, by the component i_1 of total current i_μ in winding $w_\sim$ of the first core and current i_2 in winding w_c of the second core:

$$i_1 = \frac{H_1 l}{0.4\pi w_\sim} \quad \text{and} \quad i_2 = \frac{H_2 l}{0.4\pi w_c}, \tag{2}$$

where l is the mean core length.

When inductions B_1 and B_2 vary simultaneously, the following magnetizing current must flow through winding $w_\sim$ of the first core:

$$i_\mu = i_1 - \frac{w_c}{w_\sim} i_2 = \frac{l}{0.4\pi w_\sim}(H_1 - H_2). \tag{3}$$

At the beginning of the process of induction change, $H_1 = H_c$ and $H_2 = -H_c$. Therefore, the initial value of magnetizing current is

$$i_{\mu 0} = \frac{2H_c l}{0.4\pi w_\sim}. \tag{3'}$$

With no eddy currents present, this value of i_μ is maintained during the entire process of change of induc-

tions B_1 and B_2. If the voltage $e \leq i_{\mu 0}R$, then current i in winding $w_\sim$ does not reach a value sufficient to change induction B_1, and the load circuit manifests itself as a purely active circuit, in which the current is

$$i = \frac{E_m \sin \omega t}{R}. \tag{4}$$

This occurs for $0 \leq \omega t \leq \alpha_0$, where

$$\alpha_0 = \arcsin \frac{i_{\mu 0}R}{E_m}. \tag{5}$$

For $\alpha_0 \leq \omega t \leq \alpha$, the load current $i = i_\mu$, and is defined by formula (3). After saturation of the core for $\omega t = \alpha$, the load current is again defined by formula (4). The mean value of the load current is

$$I_L = \frac{1}{\pi}\int_0^\pi i d\omega t = \frac{1}{\pi}\left(\int_0^{\alpha_0} \frac{e}{R} d\omega t + \int_{\alpha_0}^{\alpha} i_\mu d\omega t + \int_\alpha^\pi \frac{e}{R} d\omega t\right).$$

By integrating, we obtain

$$I_L = \frac{E_m}{\pi R}(2 - \cos\alpha_0 + \cos\alpha) + I_\mu, \tag{6}$$

where the mean value of the magnetizing current is

$$I_\mu = \frac{l}{0.4\pi w_\sim} \frac{1}{\pi}\int_0^\alpha (H_1 - H_2)\, d\omega t. \tag{7}$$

We note that the magnitude of α_0 is ordinarily so small that we can, in practice, take $\alpha_0 = 0$. In fact, the magnitude of $i_{\mu 0}$ in formula (5) is less than the minimum load current, $I_{L\,min} = I_{\mu(\alpha=\pi)}$. By expressing E_m in terms of the maximum value of load current, $I_{L\,max}$:

$$E_m = \frac{\pi}{2} I_{L\,max} R,$$

we find, from (5), that

$$\alpha_0 < \arcsin \frac{2 I_{L\,min}}{\pi I_{L\,max}} = \arcsin \frac{2}{\pi K_{mu}},$$

where K_{mu} is the multiplicity of load current variation.

Even for $K_{cr} = 10$, we have $\alpha_0 < 0.064$ and $\cos\alpha_0 > 0.998$. Therefore, in order to simplify the calculations, we shall henceforth take $\alpha_0 = 0$. Then, instead of (6), we obtain

$$I_L = \frac{E_m}{\pi R}(1 + \cos\alpha) + I_\mu. \tag{8}$$

We now find the expression for the average current I_c in the control circuit. In the interval $\alpha \leq \omega t \leq \pi$, the induction $B_1 = B_s$ = const, and variation of B_2 can occur only under the action of a signal voltage E_c. However, with the assumptions we have made ($R_c \to 0$), the magnitude of E_c is very small, and cannot give rise to any essential variation of B_2 in the time interval cited. We may, therefore, consider that B_2, just as B_1, varies only for $0 \leq \omega t \leq \alpha$. For $\alpha \leq \omega t \leq \pi$, current i_2, in view of the very slow change of induction B_2, cannot increase in absolute value beyond the value

$$I_L = \frac{H_c l}{0.4\pi w_c}. \tag{9}$$

Thus, in taking into account that the change in the core's magnetic state occurs along the descending arm of the hysteresis loop, we obtain for the control current:

$$I_L = \frac{1}{\pi}\left[\frac{l}{0.4\pi w_c}\int_0^{\alpha} H_2 d\omega t - I_L \int_{\alpha}^{\pi} d\omega t\right]. \tag{10}$$

Equations (8) and (10) are the amplifier's input-output characteristic.

If we ignore the voltage drop Ri_{μ} in the load circuit caused by the magnetizing current, then we can describe the variations in inductions B_1 and B_2 for $0 \leq \omega t \leq \alpha$ by the equations

$$B_1 = B_{10} + \frac{10^8}{w_{\sim}S}\int_0^t E_m \sin \omega t dt = B_{10} + kB_s(1 - \cos \omega t), \tag{11}$$

$$B_2 = B_s - \frac{10^8}{w_{\sim}S}\int_0^t E_m \sin \omega t dt = B_s - kB_s(1 - \cos \omega t), \tag{12}$$

where

$$k = \frac{\Delta B_m}{2B_s} = \frac{E_m \cdot 10^8}{\omega w_{\sim} S B_s} \tag{13}$$

is the ratio of the maximum possible induction change ΔB_m during the half-period (for $\alpha = \pi$) to twice the saturation induction. In the sequel, we shall assume that $k \leq 1$. For $\omega t = \alpha$, induction $B_1 = B_s$ and, therefore, we have from (11) that

$$B_{10} = B_s - kB_s(1 - \cos \alpha). \tag{14}$$

By substituting the values $B = B_1$ and $B = B_2$ in (1), we find the corresponding expressions for H_1 and H_2, after which we can integrate Eqs. (7) and (10):

$$I_{\mu} = \frac{l}{0.4\pi w_{\sim}}\frac{1}{\pi}\int_0^{\alpha}[2H_c + \pi^2/\gamma d^2 kB_s \cdot 10^{-9}(2 + k + k\cos\alpha - 2k\cos\omega t) \times$$
$$\times \sin \omega t]\, d\omega t = \frac{l}{0.4\pi w_{\sim}}\left[2H_c\frac{\alpha}{\pi} + 2\pi/\gamma d^2 B_s k \cdot 10^{-9}(1 - \cos\alpha)\right], \tag{15}$$

$$I_c = \frac{l}{0.4\pi w_c}\left\{-H_c\frac{\pi - \alpha}{\pi} - \frac{1}{\pi}\int_0^{\alpha}[H_c + \pi^2/\gamma d^2 k^2 B_s \cdot 10^{-9}(1 - \cos\omega t)\sin\omega t] \times\right.$$
$$\left.\times\, d\omega t\right\} = \frac{l}{0.4\pi w_c}[H_c + 5\pi/\gamma d^2 k^2 B_s \cdot 10^{-10}(1 - \cos\alpha)^2]. \tag{16}$$

If we neglect the component I_{μ} of the load current, then we can obtain, from (8) and (16), the following equation for the amplifier's input-output characteristic in relative units:

$$\frac{I_c}{I_u} = -1 - \frac{2\pi/\gamma d^2 k^2 B_s}{H_c \cdot 10^9}\left(1 - \frac{I_L}{I_{L\,max}}\right). \tag{17}$$

We note that the dynamic coercive force of a core for a sinusoidal induction with crest value $B_m = B_s$ equals* [3]

$$H_{cd} = H_c + \pi^2/\gamma d^2 B_s \cdot 10^{-9}. \tag{18}$$

*This expression can be obtained from Eq. (1) by setting B = 0.

Equation (17) may, therefore, be given in the following form:

$$\frac{I_c}{I_u} = 1 - \frac{2}{\pi} k^2 \left(\frac{H_{cd}}{H_c} - 1\right)\left(1 - \frac{I_L}{I_{L\,max}}\right)^2, \tag{19}$$

Double-Half-Wave Amplifiers with Pulse Voltage Supply

In many cases, magnetic amplifiers are supplied by virtually rectangular pulses (Fig. 5a) for the purpose of reducing the size of the amplifier for a given power. We denote by E the peak value of the supplied voltage pulses and by τ the duration of the pulses, and we then consider the operation of the circuit of Fig. 2a under the same assumptions previously made.

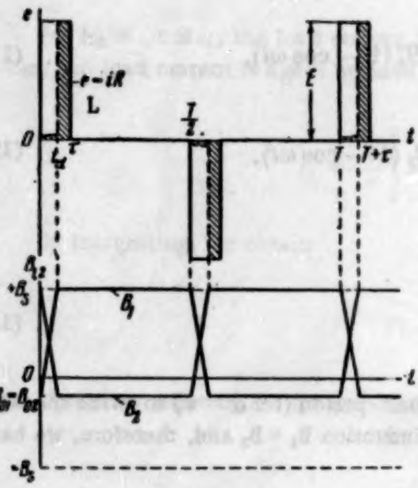

Fig. 5.

During an operating half-period, the induction B_1 of the first core is varied, under the influence of supply voltage e = E, in correspondence with the following law (Fig. 5b):

$$B_1 = B_{10} + \frac{10^8}{w_{\sim}S}\int_0^t E\,dt = B_{10} + \frac{E\cdot 10^8}{w_{\sim}S}\,t = B_{10} + 2kB_s\frac{t}{\tau}, \tag{20}$$

where B_{10} is the initial value of B_1 for t = 0 and

$$k = \frac{E\tau\cdot 10^8}{2w_{\sim}SB_s}. \tag{21}$$

Here, just as in the case of a sinusoidal voltage supply, <u>k</u> is the ratio of the maximum possible variation during one half-period of the induction of one of the cores under the influence of the supply voltage to twice the saturation induction.

For induction B_2 we have

$$B_2 = B_s - 2kB_s\frac{t}{\tau}. \tag{22}$$

For t = t_α, induction B_1 becomes equal to B_s and the first core saturates, while the load current reaches the value of

$$i = E/R. \tag{23}$$

With this, B_2 reaches the value $B_{20} = B_{10}$ and, due to the small value of the voltage signal, does not vary further until time t = T/2. Starting with t = T/2, the cores exchange roles.

The average value of current in the active load is found from the formula

$$I_L = \frac{2}{T}\int_0^{t_\alpha} i_\mu dt + \frac{2}{T}\int_{t_\alpha}^{\tau}\frac{E}{R}dt = I_\mu + \frac{2E}{R}\frac{\tau - t_\alpha}{T}. \tag{24}$$

For the values of H_1 and H_2 which define the magnitudes of the magnetizing current I_μ (7), we find, from (1), (20), and (22), that

$$H_1 = H_c + \frac{\pi\gamma d^2 k B_s}{\tau\cdot 10^9}\left(1 + \frac{B_{10}}{B_s} + 2k\frac{t}{\tau}\right), \tag{25}$$

$$H_2 = -H_c - \frac{2\pi\gamma d^2 k^2 B_s t}{\tau^2 \cdot 10^9}. \tag{26}$$

By taking into account that

$$B_{10} = B_{20} = B_s - 2kB_s \frac{t_\alpha}{\tau}, \tag{27}$$

we find that

$$I_\mu = \frac{l}{0.4\pi w_\sim} \frac{2}{T} \int_0^{t_\alpha} (H_1 - H_2)\,dt = \frac{l}{0.4\pi w_\sim} \left[H_c \frac{4t_\alpha}{T} + \frac{4\pi\gamma d^2 k B_s}{T \cdot 10^9} \frac{t_\alpha}{\tau} \right]. \tag{28}$$

For a constant component of control current we find, by analogy with (10), that

$$I_c = \frac{l}{0.4\pi w_c} \frac{2}{T} \left[\int_0^{t_\alpha} H_2 dt - \int_{t_\alpha}^{T/2} H_c dt \right] = \frac{l}{0.4\pi w_c} \left(-H_c - \frac{2\pi\gamma d^2 k^2 B_s}{T \cdot 10^9} \frac{t_\alpha^2}{\tau^2} \right). \tag{29}$$

If we neglect component I_μ of the load current then, from (24), (29), and (9), we can obtain the amplifier's input-output characteristic equation

$$\frac{I_c}{I_u} = -1 - \frac{2\pi\gamma d^2 k^2 B_s}{T H_c \cdot 10^9} \left(1 - \frac{I_L}{I_{L\,max}} \right)^2, \tag{30}$$

where

$$I_{L\,max} = \frac{2E}{R} \frac{\tau}{T}. \tag{31}$$

By taking into account that $T = 1/f$, we find from (17) and (30) that the input-output characteristics of amplifiers with the circuits of Fig. 2 coincide (to within the magnetizing current I_μ in the load circuit) when the amplifiers are supplied with sinusoidal voltages or with voltage pulses.

Double Half-Wave Fast-Acting Magnetic Amplifiers

We now consider fast-acting magnetic amplifiers, based on the Ramey circuit (Fig. 3), for the case when the signal voltage e_c is a double-half-wave rectified (unsmoothed) sinusoidal voltage with crest value E_c, either coinciding in phase or 180° out of phase with the supply voltage (Fig. 6). At the same time as the line voltage is being applied to working winding $w_\sim$ of one of the cores, via the corresponding diode, the second core is "demagnetized" by the action of the total voltage

$$e_2 = E'_m \sin \omega t - E_c \sin \omega t, \tag{32}$$

which is applied to its w_c winding. Here, $E'_m = E_m w_c / w_\sim$. The change in induction in this core during one controlling half-period is defined by the formula

$$B_2 = B_s - \frac{10^8}{w_c S} \int_0^t e_2 dt = B_s - kB_s \left(1 - \frac{E_c}{E'_m} \right) (1 - \cos \omega t), \tag{33}$$

where formula (13) holds for k. At the end of the controlling half-period ($\omega t = \pi$) we get

$$B_{20} = B_{10} = B_s - 2kB_s \left(1 - \frac{E_c}{E'_m} \right). \tag{34}$$

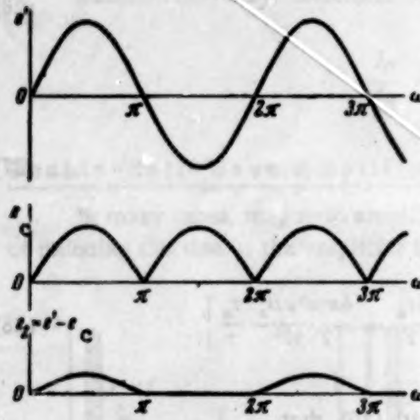

Fig. 6.

As before, the load current is defined by formula (8) with the sole difference that the magnetizing current now equals

$$I_\mu = \frac{l}{0.4\pi w_\sim}\,\frac{1}{\pi}\int_0^\alpha H_1\,d\omega t, \tag{35}$$

i.e., the supply of the working circuit in the circuit of Fig. 3, unlike the circuits of Fig. 2, attains, in the interval $0 \le \omega t \le \alpha$, the magnetizing current I_μ necessary for changing the induction of only one of the cores from B_{10} to B_s. Therefore, the magnitude of I_μ in this circuit is less than for the circuits of Fig. 2.

Since Eq. (14) is valid for the amplifier now under consideration then, by taking (34) into account, we find that

$$\cos\alpha = 2\frac{E_c}{E'_m} - 1$$

and the load current (8) can be expressed in the following way:

$$\frac{I_L}{I_{L\,max}} = \frac{E_c}{E'_m} + \frac{I_\mu}{I_{L\,max}}. \tag{36}$$

From (1) and (33), we now find the expression for the field strength H_2 induced by current i_2 through the signal source:

$$H_2 = -H_c - \pi^2 f\gamma d^2 k^2 B_s\cdot 10^{-9}\left(1 - \frac{E_c}{E'_m}\right)^2 (1 - \cos\omega t)\sin\omega t.$$

By taking into consideration that, in the amplifier we are discussing, the induction B_2 varies during the entire controlling half-period, we find the following formula for the constant component of the control current:

$$I_c = \frac{l}{0.4\pi w_c}\,\frac{1}{\pi}\int_0^\pi H_2\,d\omega t = \frac{l}{0.4\pi w_c}\left[-H_c - 2\pi f\gamma d^2 k^2 B_s\cdot 10^{-9}\left(1 - \frac{E_c}{E'_m}\right)^2\right]. \tag{37}$$

If we ignore the second component of the load current in formula (36), which is due to the magnetizing current for one of the cores, we obtain, from (36) and (37), exactly the same Eq. (17) for the input-output characteristic of the fast-acting magnetic amplifier of Fig. 3 as we obtained earlier for the ordinary double-half-wave amplifiers of Fig. 2.

Double-Half-Wave Amplifiers with an Arbitrary Form of Supply Voltage Curve

We now show that the coincidence observed above of the input-output characteristics of amplifiers for different laws of variation of induction B_2 during the controlling half-periods is not of a random character, but is also valid for any form of supply voltage curve, $e = e(t)$.

Assuming that, for $t = 0$ and $t = T/2$, voltage $e = 0$, and that $e(T+t) = e(t)$, we denote, as before,

$$\int_0^{T/2} e(t)\,dt = k2w_\sim SB_s\cdot 10^{-8} \qquad (k < 1). \tag{38}$$

If the first core in the circuit of Fig. 2a is saturated for $t = t_\alpha$ then, for the average value of load current, neglecting the magnetizing current which flows for $0 \le t \le t_\alpha$, we obtain the expression

$$I_L = \frac{1}{R}\frac{2}{T}\int_{t_\alpha}^{T/2} e(t)\,dt. \tag{39}$$

For $t_\alpha = 0$, the load current attains its maximum value:

$$I_{L\,max} = \frac{4kw_\sim SB_s}{RT\cdot 10^8}, \tag{40}$$

For $0 \leq t \leq t_\alpha$, inductions B_1 and B_2 vary in accordance with the following laws:

$$B_1 = B_{10} + \frac{10^8}{w_\sim S}\int_0^t e(t)\,dt, \qquad B_2 = B_s - \frac{10^8}{w_\sim S}\int_0^t e(t)\,dt \tag{41}$$

For t = 0, we have $B_1(0) = B_{10}$ and $B_2(0) = B_s$ and, for $t = t_\alpha$, $B_1(t_\alpha) = B_s$ and $B_2(t_\alpha) = B_{20} = B_{10}$. Since

$$\int_{t_\alpha}^{T/2} e(t)\,dt = \int_0^{T/2} e(t)\,dt - \int_0^{t_\alpha} e(t)\,dt,$$

we then obtain, from (38)-(41), that

$$I_L = I_{L\,max} - \frac{2w_\sim S\cdot 10^{-8}}{RT}(B_s - B_{10}). \tag{42}$$

We now find the magnitude of the control current. By setting $B = B_2$ in Eq. (1) and integrating, we find that

$$\int H_2 dt = -H_c t + 5\pi\gamma d^2\cdot 10^{-10}\left(B_2 - \frac{B_2^2}{2B_s}\right).$$

Therefore,

$$I_C = \frac{l}{0.4\pi w_C}\frac{2}{T}\left[\int_0^{t_\alpha} H_2 dt - \int_{t_\alpha}^{T/2} H_c dt\right] = \frac{l}{0.4\pi w_C}\left[-H_c - \frac{5\pi\gamma d^2}{TB_s\cdot 10^{10}}(B_s - B_{10})^2\right]. \tag{43}$$

By eliminating $B_s - B_{10}$ from Eqs. (42) and (43), and by taking into account that $T = 1/f$, we obtain exactly the same Eqs. (17) and (30) for the circuits of Fig. 2 as were obtained when the amplifier was supplied with sinusoidal and pulse voltages.

Experimental Data

For the experimental verification of the basic theoretical results obtained above, it was necessary to have cores with rectangular static hysteresis loops for which the relative influence of magnetic viscosity and inhomogeneity of properties along the cross section would be negligibly small in comparison with the influence of eddy currents. For this purpose, cores made of alloy 65NP, 0.35 mm thick, were used for the experimental amplifier After annealing in a magnetic field, the cores had a coefficient of rectangularity (i.e., the ratio of the residual induction B_r to the induction B_s in a field B = 5 oe) exceeding 0.99 for a coercive force of $H_c = 0.025$ oe. The cores and the amplifier had the following basic parameters: d = 0.35 mm, l = 78 mm, S = 31.5 mm², $\gamma = 4\cdot 10^4$ ohm cm^{-1}, B_s = 13000 gauss, f = 50 cps, $w_C = w_\sim$ = 1200 turns, R = 150 ohm, and k = 0.9. Specially prepared plane germanium diodes were used in the amplifier in order to eliminate the effect of feedback currents on the amplifier's characteristics. The function $I_L/I_{L\,max} = f_1(I_C)$ was computed by formula (17) without taking magnetizing current I_μ into account. This function is shown by curve 1 on Fig. 7. Curve 2 in this figure represents the function $I_\mu/I_{L\,max} = f_2(I_C)$, computed from formulas (8), (15), and (17). By summing the ordinates of curves

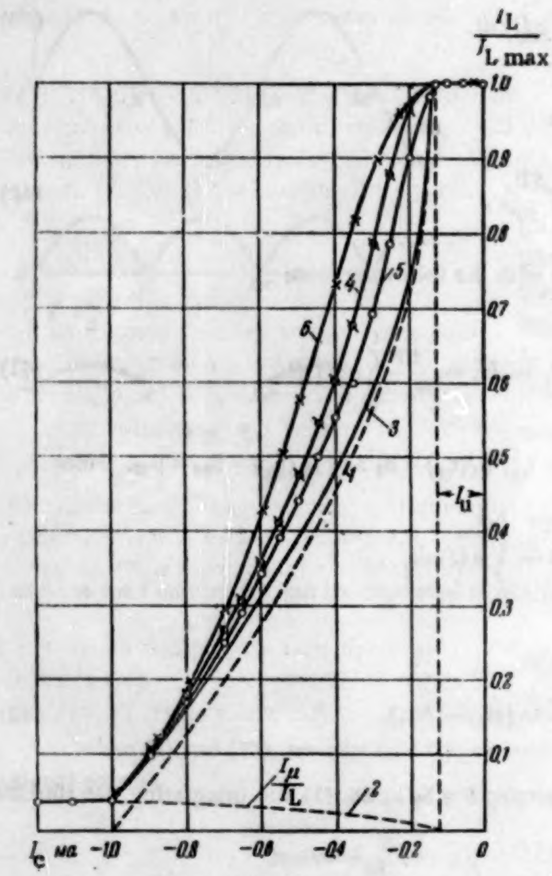

Fig. 7.

1 and 2, we obtain the computed input-output characteristic of the amplifier with magnetizing current being taken into account (curve 3 of Fig. 7). Curves 4 and 5 of Fig. 7 are experimental characteristics, obtained for the circuit of Fig. 2b when the amplifier was supplied by a sinusoidal voltage (curve 4) and by rectangular pulses (curve 5). The experimental characteristics for the other circuits of Fig. 2 virtually coincided with the characteristics obtained for the circuit of Fig. 2b.

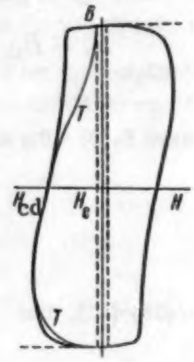

Fig. 8.

The curves given attest to a satisfactory coincidence of theoretical and experimental characteristics, particularly in the region of small load currents. The discrepancies observed in the region of large load currents between the theoretical and experimental characteristics, and also between the experimental characteristics corresponding to sinusoidal and to pulse-supply voltages have a completely regular character, and are explained by the effect of magnetic viscosity.

Figure 8 shows the oscillogram of the dynamic hysteresis loop of one of the cores for a sinusoidal variation of the induction with a frequency of 50 cps. The dashed lines represent the static hysteresis loop, obtained by the ballistic method. The light lines, denoted by the letter T, correspond to the theoretical dynamic hysteresis loop computed from formula (1). It is obvious that, when the sign of the field strength changes, there is a noticeable lag in the variation of the induction in comparison with theoretical curve T, this lag not being caused by eddy currents, since the induction is practically invariant with this. Essential differences of the experimental dynamic hysteresis loop from the theoretical one are only observed at the initial portion of the descending (or ascending) arm of the hysteresis loop, which also explains the character of the deviations of the theoretical curves of Fig. 7 from the experimental ones.

The relative effect of viscosity, as compared to that of eddy currents, will be the greater, the slower the variation of magnetic flux. When an amplifier is supplied by a sinusoidal voltage, the speed of flux variation in the controlling half-period, with large load currents, is significantly less than for an amplifier supplied by rectangular voltage pulses. Therefore, the effect of viscosity is greater in the first case, and the upper portion of the amplifier's input-output characteristic (4) is shifted to the left in the region of large absolute values of the control current (Fig. 7).

In the fast-acting amplifier of Fig. 3, the variation of induction in the controlling half-period occurs even more slowly than in the ordinary circuits of Fig. 2. This is explained by the fact that the induction varies from B_s to $B_{2\theta}$ during the entire controlling half-period in the fast-acting circuit, whereas this occurs in the circuits of Fig. 2 only during the interval $0 \leq \omega t \leq \alpha$. We should, therefore, expect a more significant viscosity effect in the fast-acting circuit, which is attested to by the input-output characteristic of this circuit (curve 6 of Fig. 7).

In the region of small load currents, when the duration and speed of magnetic flux variation in the controlling half-period are practically identical for the circuits of Figs. 2 and 3, the characteristics of these circuits also coincide.

It is interesting to note that the presence of viscosity can lead to an improvement in the rectangularity of the dynamic hysteresis loop, an improvement in the linearity of the input-output characteristic and an increase in its steepness. As the frequency of the supply voltage is increased, the relative effect of the eddy currents increases, and a closer coincidence of theoretical and experimental characteristics is observed for all the circuits.

SUMMARY

In this work we considered theoretically the effect of eddy currents on the characteristics of the basic types of magnetic amplifiers with feedback for very small impedances of the control circuit and with rectangular hysteresis loops. Analytical expressions were obtained for the input-output characteristics of magnetic amplifiers with eddy currents present. It was established that their quantitative effect is the same for all types of amplifiers considered (to within the component of load current due to the magnetizing current of the unsaturated core), independently of the form of the supply voltage curve.

The experimental characteristics, obtained for a test magnetic amplifier, were found to be in sufficiently close agreement with the theoretical ones. With this, for 0.35 mm thick alloy 65NP, even for a supply frequency of 50 cps, there is a very significant effect of eddy currents on the characteristics of magnetic amplifiers with feedback.

A significant effect of magnetic viscosity on magnetic amplifier input-output characteristics was observed, even for a marked development of a surface effect. The effect of viscosity, as distinguished from that of eddy currents, depends on the amplifier circuit and on the form of the supply voltage. It is the stronger, the more slowly the variation of magnetic flux occurs in the controlling half-period.

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ON THE QUESTION OF DETERMINING THE OPTIMAL COEFFICIENTS OF IMPULSIVE RESPONSES

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A formula is presented for determining the impulsive response of a system which will be optimal for "white noise."

In the case of an input signal consisting of a controlling action

$$g(t) = \sum_{i=0}^{r} g_0^{(i)} \frac{t^i}{i!} \tag{1}$$

and a disturbance in the form of "white noise," whose spectral density we denote by S_0, the system's optical impulsive response is defined by the expression [1, 2]

$$K(t) = \frac{1}{S_0} \sum_{i=0}^{r} \gamma_i t^i. \tag{2}$$

The coefficients γ_i of expression (2) are found by solving the system of r+1 equations

$$\frac{1}{S_0} \sum_{i=0}^{r} \gamma_i \frac{T^{i+1+m}}{i+1+m} = \mu_m \qquad (m = 0, 1, \ldots, r), \tag{3}$$

where the μ_m are forces which we will consider as given.

The solution of system (3) of equations can be expressed in terms of determinants

$$\gamma_i = \sum_{m=0}^{r} \mu_m \frac{A_{mi}}{D} \qquad (i = 0, 1, \ldots, r), \tag{4}$$

where D is the determinant of the system and the A_{mi} are the co-factors of the corresponding elements.

The first subscript of a co-factor corresponds to the number of the row in the determinant (the numeration starts with zero) and the second subscript corresponds to the column.

The terms $\mu_m A_{mi}/D$ for $\mu_m = (-1)^m m!$ which enter into expression (4) are the coefficients γ_i of the optimal m-differentiator filter which is compensated for the first r−m derivatives.

The solution of system (4) of equations is quite arduous for $r > 3$. We propose the following formula for determining the ratios A_{mi}/D:

TABLE 1

r	γ_0		$-\gamma_1$		γ_2		$-\gamma_3$		γ_4		$-\gamma_5$
0	1		—		—		—		—		—
	$\left(\frac{2}{1}\right)^2$										
1	4	$\frac{3}{2}$	6		—		—		—		—
	$\left(\frac{3}{2}\right)^2$										
2	9	$\frac{3+5}{2}$	36	$\frac{5}{2+4}$	30		—		—		—
	$\left(\frac{4}{3}\right)^2$										
3	16	$\frac{3+5+7}{2}$	120	$\frac{5+7}{2+4}$	240	$\frac{7}{2+4+6}$	140		—		—
	$\left(\frac{5}{4}\right)^2$										
4	25	$\frac{3+5+7+9}{2}$	300	$\frac{5+7+9}{2+4}$	1050	$\frac{7+9}{2+4+6}$	1400	$\frac{9}{2+4+6+8}$	630		—
	$\left(\frac{6}{5}\right)^2$										
5	36	$\frac{3+5+7+9+11}{2}$	630	$\frac{5+7+9+11}{2+4}$	3360	$\frac{7+9+11}{2+4+6}$	7560	$\frac{9+11}{2+4+6+8}$	7560	$\frac{11}{2+4+6+8+10}$	2772

TABLE 2

r	γ_1		$-\gamma_2$		γ_3		$-\gamma_4$		γ_5		$-\gamma_6$
1	6	$\frac{6}{3}$	12		—		—		—		—
	$\frac{6}{1}$										
2	36	$\frac{6+10}{3}$	192	$\frac{15}{3+13}$	180		—		—		—
	$\frac{6+4}{1+2}$										
3	120	$\frac{6+10+14}{3}$	1200	$\frac{15+21}{3+13}$	2700	$\frac{28}{3+13+29}$	1680		—		—
	$\frac{6+4+5}{1+2+3}$										
4	300	$\frac{6+10+14+18}{3}$	4800	$\frac{15+21+27}{3+13}$	18900	$\frac{28+36}{3+13+29}$	26880	$\frac{45}{3+13+29+51}$	12600		—
	$\frac{6+4+5+6}{1+2+3+4}$										
5	630	$\frac{6+10+14+18+22}{3}$	14700	$\frac{15+21+27+33}{3+13}$	88200	$\frac{28+36+44}{3+13+29}$	211680	$\frac{45+55}{3+13+29+51}$	220500	$\frac{66}{3+13+29+51+79}$	83160

$$\frac{A_{mi}}{D} = (-1)^{i+m} \frac{A_{r+m+1}^{2m+1}}{(m!)^2} \frac{A_{r+1+i}^{2i+1}}{(i+m+1)(i!)^2} \frac{S_0}{T^{i+m+1}}, \tag{5}$$

where a term of the form A_p^q denotes the number of ways of placing p things taken q at a time, i.e., the variation (permutations) of p things taken n at a time.

Expression (5) allows the following formula to be obtained for determining the coefficients γ_i:

$$\gamma_i = S_0 \sum_{m=0}^{r} (-1)^{i+m} \mu_m \frac{A_{r+m+1}^{2m+1}}{(m!)^2} \frac{A_{r+1+i}^{2i+1}}{(i+m+1)(i!)^2 T^{i+m+1}} \quad (i = 0, 1, \ldots, r). \tag{6}$$

Formula (5) was obtained inductively by considering the solution of system (3) of equations for a number of values of r and for m = 0,1,2. Table 1 gives the numerical values of the coefficients γ_i (S_0 and T are omitted) for the optimal filters ($\mu_0 = 1$; $\mu_1 = \ldots = \mu_r = 0$). Table 2 gives the values of γ_i for the optimal differentiator filters ($\mu_1 = -1$; $\mu_0 = \mu_2 = \ldots = \mu_r = 0$). In both tables, between neighboring values of γ_0 and also γ_i and γ_{i+1}, there are given the ratios of the following coefficient to the preceding one. A consideration of these ratios allowed a number of regularities to be remarked, for which the numerators and the denominators of the ratios are written in the form of sums of regularly varying terms. The result of generalizing these regularities is formula (5).

We consider two examples of the determination of the coefficients γ_i by formula (6).

Example 1. To determine the coefficients of the impulsive response, optimal for "white noise," of a double differentiator filter compensated by the first derivative.

We have that r = 3, m = 2, $\mu_2 = 2!$, $\mu_0 = \mu_1 = \mu_3 = 0$. Formula (6) takes the form:

$$\gamma_i = (-1)^{i+2} \frac{A_6^5}{2!} \frac{A_{4+i}^{2i+1}}{(i+3)(i!)^2} \frac{S_0}{T^{i+3}}.$$

From this we easily obtain the coefficients $\gamma_0 = 480\ S_0/T^3$, $\gamma_1 = -5400\ S_0/T^4$, $\gamma_2 = 12960\ S_0/T^5$, and $\gamma_3 = -8400\ S_0/T^6$, which satisfy the given forces.

Example 2. To determine the coefficients of the impulsive response, optimal for "white noise," of the uncompensated triple differentiator filter.

Here, r = 3, m = 3, $\mu_3 = -3!$, $\mu_0 = \mu_1 = \mu_2 = 0$. Formula (6) takes the form:

$$\gamma_i = -(-1)^{i+3} \frac{A_7^7}{3!} \frac{A_{4+i}^{2i+1}}{(i+4)(i!)^2} \frac{S_0}{T^{i+3}}.$$

From this we easily obtain the coefficients $\gamma_0 = 840\ S_0/T^4$, $\gamma_1 = -10080\ S_0/T^5$, $\gamma_2 = 25200\ S_0/T^6$, $\gamma_3 = -16800\ S_0/T^7$, which satisfy the given forces.

SUMMARY

The formula presented allows one to determine easily the coefficients γ_i of the impulsive response, optimal for "white noise," which satisfies given forces. This same formula can also be used for solving systems of linear algebraic equations whose numerical coefficients, reading from right to left and from top to bottom, comprise the number series $^1/_1$, $^1/_2$, $^1/_3$, $^1/_4$, etc.

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ON SETTING UP AN OPERATING PROGRAM FOR SEVERAL CONTROL SYSTEMS

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Frequently, a dispatcher or operator has to observe a significant number of quantities which characterize a process being controlled. For such observations there have recently been developed the so-called "run-around devices" which permit successive observation of the various parameters of the controlled process. However, in the observation of various quantities during equal intervals of time, there is the danger that some rapidly varying quantity will go beyond its permissible limits before "its turn" comes around. Conversely, a slowly varying quantity will be observed too frequently.

We present below a method for improving the quality of operation of such systems by setting up a program based on probability relationships.

Let n quantities be observed. We shall assume that, from the probabilistic point of view, these quantities are independent, or are weakly dependent. In fact, if there existed a functional dependence $X_k = f(X_i)$, we could control quantity X_k by controlling quantity X_i.

Let the signal from quantity X_i be applied to the control or inspection devices during time interval t_i.

We shall assume that the system signals deviations of the observed quantities beyond definite limits. For the sake of generality, we shall assume that the deviations which give rise to signals are asymmetrically distributed about the quantity's optimal value X_{i0}, and we shall denote them by $\delta X'_i$ and $\delta X''_i$.

In addition, let the maximum admissible limits of deviations of quantity X_i be A'_i and A''_i. Then X_i will lie within the limits

$$X_{i0} - A'_i < X_i < X_{i0} + A''_i.$$

The quantities A_i are given by the conditions of satisfactory operation of the controlled object; in particular, they might be given by the conditions for accident-free or danger-free operation.

If, at time t = 0, the quantity X_i equaled X_{i0}, then its possible values during time interval t are conditioned by the probability distribution function of the random variable X_i.

The probability that, with observation on regulation of quantity X_i during time t, this quantity goes beyond the admissible limits is [1]:

$$p^{(t)} = p^{(t)}_{(A'_i,\, A''_i)} + p^{(t)}_{(\delta X'_i,\, \delta X''_i)}\, p^{(2t)}_{(A'_i,\, A''_i)} + \\ + p^{(t)}_{(\delta X'_i,\, \delta X''_i)}\, p^{(2t)}_{(\delta X'_i,\, \delta X''_i)}\, p^{(3t)}_{(A'_i,\, A''_i)} + \ldots \tag{1}$$

Here, $p^{(t)}_{(A'_i, A''_i)}$ is the probability that the magnitude of X_i goes beyond the admissible limits before time t, $p^{(t)}_{(\delta X'_i, \delta X''_i)}$ is the probability that the deviations of X_i before time t will be less than the deviations giving

rise to signals, $P^{(2t)}_{(A'_i, A''_i)}$ is the probability that quantity X_i goes out of bounds before time 2t, etc.

We now elucidate formula (1). The circumstance that X_i can go out of bounds before time t is expressed in formula (1) by the term $P^{(t)}_{(A'_i, A''_i)}$. However, during this same time, quantity X_i can vary so slightly that no signal is forthcoming from it. In this case, X_i will not be regulated, and will continue to vary independently of the controller during time 2t. This case is expressed by the term $P^{(t)}_{(\delta X'_i, \delta X''_i)} P^{(2t)}_{(A'_i, A''_i)}$, which is the probability that the quantity X_i can go out of bounds during time 2t if it were not regulated at time t.

Since probability P is less than one, the terms of the series will decrease rapidly. Therefore, for practical computations, we can limit ourselves to several of the first terms only.

Now, let the maximum allowable probability that quantity X_i go out of bounds be given. We denote it by P_{max}.

The magnitude of P_{max} depends on the nature of the quantity being controlled and its role in the process being controlled. The more heavily significant deviations of X_i from X_{i0} reflect themselves on the operation of the object being controlled, the smaller must P_{max} be. Thus, the determination of P_{max} depends on the properties of the object and its individual parameters.

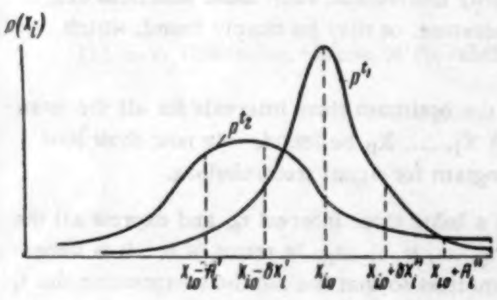

Fig. 1.

It is obvious that $P^{(t)}$ must be less than P_{max}. As the time interval t increases, the magnitude of $P^{(t)}$ increases. For some t, the equality $P^{(t)} = P_{max}$ will hold. This t will also be the optimal control time t_i. Indeed, increasing t makes it possible to devote a larger segment of time to the control process. On the other hand, the increase of t_i is limited by P_{max}.

The method of determining t_i is best illustrated by examples.

For the quantity X_i, let the probability distribution density be given in the form of curves, constructed for several times t (Fig. 1).

Here, $p^{(t_1)}$ is the curve of the probability distribution density for time interval t_1, and $p^{(t_2)}$ is the analogous curve for time t_2. Then,

$$P^{(t)}_{(A'_i,\, A''_i)} = 1 - \int_{X_{i0}-A'_i}^{X_{i0}+A''_i} p^{(t)}(X_i)\, dX_i, \qquad (2)$$

$$P^{(t)}_{(\delta X'_i,\, \delta X''_i)} = \int_{X_{i0}-\delta X'_i}^{X_{i0}+\delta X''_i} p^{(t)}(X_i)\, dX_i, \qquad (3)$$

$$P^{(2t)}_{(A'_i,\, A''_i)} = 1 - \int_{X_{i0}-A'_i}^{X_{i0}+A''_i} p^{(2t)}(X_i)\, dX_i. \qquad (4)$$

The integrals in formulas (2)-(4) are easily computed graphically from the areas bounded by the curves $p(X_i)$.

For the determination of t_i, it is necessary to find, on the basis of (1) and (2)-(4), the value of P^t for several t. With P_{max} known, the magnitude of t_i can be found by interpolation between the values of t_i, to which the probabilities $P^{(t)}$ correspond, one of which is greater than P_{max} and the other smaller.

If the probability distribution densities are not given for the random variables X_i, then one can quite simply determine the probabilities from the graphs of the distribution functions themselves (Fig. 2):

$$p^{(t)}_{(A_i', A_i'')} = 1 - [F_t(X_{i0} + A_i'') - F_t(X_{i0} - A_i')]. \tag{5}$$

$$p^{(t)}_{(\delta X_i', \delta X_i'')} = [F_t(X_{i0} + \delta X_i'') - F_t(X_{i0} - \delta X_i')]. \tag{6}$$

$$p^{(2t)}_{(A_i', A_i'')} = 1 - [F_{2t}(X_{i0} + A_i'') - F_{2t}(X_{i0} - A_i')]. \tag{7}$$

where F_t and F_{2t} are the ordinates of the probability distribution curves given, respectively, for t and 2t.

It should be mentioned that it is completely nonobligatory to know the probability distribution density of the probability distribution function. In practice, it is necessary to construct, from experimental data, the graphs of several probabilistic characteristics of the quantities X_i for various times t and, by interpolation, to find the optimal times t_i. Expressing the probabilistic characteristics by functions is only convenient when these functions are known in the literature, or may be simply found, which rarely happens.

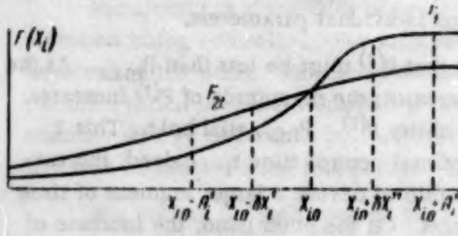

Fig. 2.

Thus, let the optimum time intervals for all the quantities $X_1, X_2, \ldots, X_i, \ldots, X_n$ be found. We now show how one sets up a program for signal transmissions.

We select a basic time interval t_0, and express all the other times, $t_1, t_2, \ldots, t_i, \ldots, t_n$, in terms of it. It is necessary to arrange matters so that the numbers expressing the t_i in terms of t_0 will be mutually prime. With this, the first part of the program setup terminates.

We illustrate what has just been said by means of an example. Let there be three quantities to be regulated: X_1, X_2, and X_3. For them we find the optimal intervals: $t_1 = 3$ sec, $t_2 = 8$ sec, and $t_3 = 5$ sec. Consequently, the first quantity must be sent every 3 sec, the second every 8 sec, and the third every 5 sec.

Using this same example, we now consider the second part of the program setup.

Thus, it is necessary to send quantity X_1 at times 3 sec, 6 sec, etc., quantity X_2 at times 8 sec, etc., and quantity X_3 at times 5 sec, etc. If, at time t = 8 sec, we were to recommence the program at the beginning then, between transmissions of X_1, there would be the time 2 + 3 = 5 sec (instead of 3 sec) and, between transmissions of X_3, time 5 + 3 = 8 sec (instead of 5 sec).

If, now, the transmission of the quantities were programmed in accordance with the table

Moment of time, sec	3	5	6	8	9	10	12	15	16	17
Quantity sent	X_1	X_3	X_1	X_2	X_1	X_3	X_1	X_3	X_1	X_2

and, after this, the program were recommenced, then X_3 would be sent after 7 sec, X_1 after 4 sec, and X_2 after 8 sec.

As is clear from the example, it is desirable so to choose the moment of transition to the next cycle of the program that both the duration of the cycle, t_{end}, and the duration of the neighboring steps, $t_{end} - t_0$, $t_{end} + t_0$ (and, in case of a large number of parameters, also $t_{end} - 2t_0$, etc.), be divided properly into cycles expressing the durations of the intervals t_i. With this, the transition error is decreased. If t_{end} is made directly of several t_i's, it is then necessary to disperse the moments of transmission of the quantities.

With this, of course, attention must be given to the physical nature of the X_i. If an X_i must be maintained accurately (for example, so as not to destroy the controlled object), it is then impossible to increase the corresponding t_i, and some other t must be increased.

Technologically, this program can be implemented by means, for example, of a stepping switch with equal, or unequal, steps.

Choosing optimal time intervals and introducing them into the regulation or observation program has significant advantages.

If, as is ordinarily done, all quantities are checked successively with equal time intervals, then all the Δt's are equal among themselves. Here, Δt is the interval between signal transmissions from different quantities. Let T be the time of one observation cycle, and let us know the minimum admissible time intervals for the most rapidly varying i-th quantities. Then, $T \leq t_i$, and $\Delta t = T/n$.

If the optimum time intervals are found and are included in the program, then $\Delta t \geq t_\theta$.

Let us consider an example. Let five quantities be observed, where $t_{i\,min} = 2$ sec and $t_\theta = 1$ sec. Then, for the ordinary method of sending signals, $\Delta t = 0.4$ sec and, for the method proposed here, $\Delta t = 1$ sec.

Thus, the method proposed here makes it possible to decrease the frequency of switching. It also eases the work of the operator and increases the length of service of the apparatus.

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AN INSTRUMENT FOR DETERMINING THE FREQUENCY CHARACTERISTICS OF NONLINEAR SYSTEMS

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The theory of operation, as well as the device itself, are described for a new electronic instrument which permits the aforementioned frequency characteristics to be determined with high accuracy in the frequency range $f = 0.25$ to 50 cps, directly in the experimental process.

For the theoretical designs and dynamics simulations of automatic control systems, an ever increasing use is being made of the experimental determination of the frequency characteristics of the systems themselves and of their individual elements. However, whereas the methodology of experimental determination of the frequency characteristics of linear systems is developed in great detail, and many works devoted to this question have been published, there is still no satisfactory method for determining experimentally the frequency characteristics of

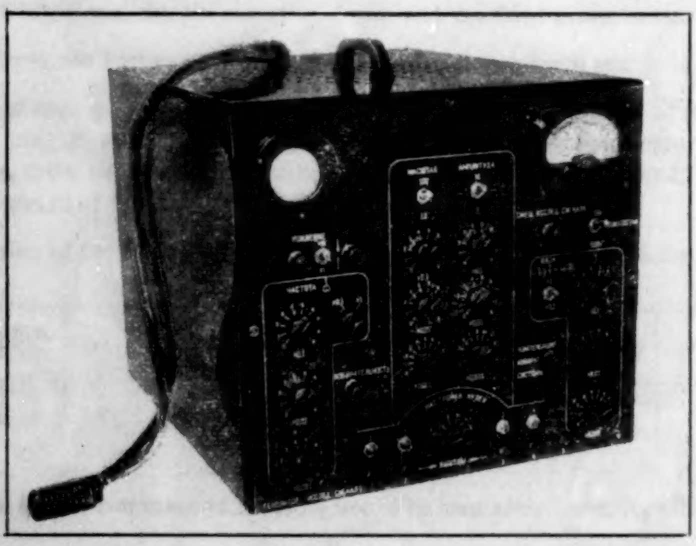

Fig. 1.

nonlinear systems which, to a definite degree, inhibits the use of frequency characteristics for the analysis of nonlinear systems. The difficulty which arises in the experimental determination of the frequency characteristics of nonlinear systems amounts to this, that the forced oscillations of their output signals due to a harmonic stimulus $u_0 = A_0 \sin \omega t$ at the input contains, not only the fundamental frequency, but also higher harmonics, and may be given in the form

$$u = a + \sum_{n=1}^{\infty} A_n \sin(n\omega t - \psi_n).$$

Since, as the frequency characteristics of nonlinear systems, the amplitude and phase characteristics of the first harmonic, $u_1 = A_1 \sin(\omega t - \psi_1)$, are used (cf. for example, [1, 2]), the problem reduces to the determination of the amplitude A_1 and phase ψ_1 of the first harmonic.

In conjunction with this, the necessity arises for estimating, at least roughly, the higher harmonics of the output signal, so that one may judge the possibilities of using the frequency characteristics found for the analysis of the dynamics of a system which contains the nonlinear element investigated, and also so that one may gauge the possibilities of linearizing the equations of motion of these nonlinear elements [3, 4].

Although the estimates cited can, in theory, be given by means of the well-known methods of harmonic analysis [5, 6], the practical use of these methods, which allow both the amplitude and the phase spectra to be determined, is arduous since, in addition to the large amount of computational work required by these methods, a preliminary recording (oscillographs) of the process under investigation is also required, which reduces the accuracy and speed of obtaining results.

Recently, attempts have been made to create special instruments for determining the frequency characteristics of nonlinear systems. In particular, the electronic instrument described in [7] permits the experimental determination of the amplitude and phase of the first harmonic of a periodic signal in the frequency range $f < 1$ cps, by automatically calculating the first coefficients in the Fourier series.

We give below a description of an electronic instrument, developed by the authors, by means of which one can, without resorting to oscillographs and complicated computations, carry out the harmonic analysis of the output signal of a nonlinear servo-system whose input and output signals are in the form of electrical voltages in the frequency range $f = 0.25$ to 50 cps. The instrument described can be used, in particular, for taking off the frequency characteristics of servo-systems and also of power-drives of automatic control systems. The over-all view of the instrument is given in Fig. 1.

Theory of Operation of the Instrument

The theory of operation of the instrument is based on the compensation (of the harmonic being investigated) of the nonlinear system's output signal $u_n = A_n \sin(n\omega t - \psi_n)$ by a specially formed signal $u^* = A^* \sin(n\omega t - \psi^*)$ whose amplitude and phase are separately controlled, and whose frequency equals the frequency of the harmonic being investigated. The fact of coincidence of the amplitude and phase of the compensating signal u^* with the

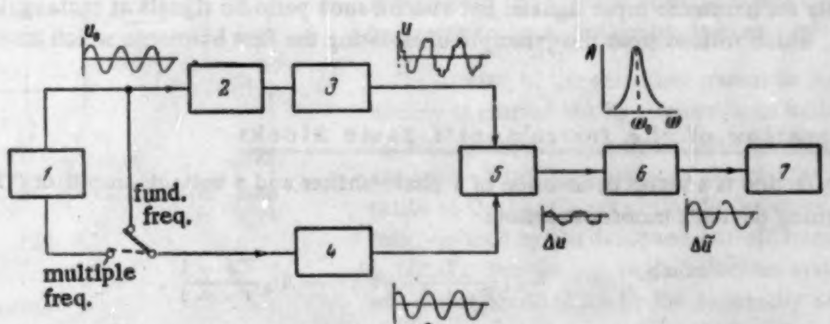

Fig. 2. 1) Sine wave generator; 2) nonlinear system under investigation; 3) scale block; 4) forming device; 5) comparison block; 6) controlled selection system; 7) indicator.

amplitude and phase of the investigated harmonic is established by the absence, in the difference $\Delta u = u - u^*$, of the harmonic of frequency $n\omega$, which is accomplished by applying the signal Δu to the input of a controlled selection system which is tuned to the frequency $n\omega$ of the harmonic being investigated, and which has a sufficiently narrow pass band.

Let us consider the operation of the instrument in the case when one wishes to determine the amplitude and phase of the fundamental harmonic of a nonlinear system's output signal (the block schematic of the device is shown in Fig. 2). The electrical signal $u_0 = A_0 \sin \omega t$ from the sine wave generator is applied simultaneously to the input of the nonlinear system being investigated and to the input of the forming device. In the forming device, by means of a block for tuning the phase and a block for tuning the amplitude, there are implemented individual transformations of the phase and the amplitude, as well as variations in the sign of the signal u_0. The output signal of the forming device, $u^* = A^* \sin(\omega t - \psi^*)$ and the investigated signal $\underline{u}$, amplified by the scale block, are applied to the comparison block.

After this, the output signal of the comparison block

$$\Delta u = u - u^* = A_\Delta \sin(\omega t - \psi_\Delta) + \sum_{n=2}^{\infty} A_n \sin(n\omega t - \psi_n),$$

where

$$A_\Delta = A_1 \sqrt{1 - 2\frac{A^*}{A_1}\cos(\psi_1 - \psi^*) + \left(\frac{A^*}{A_1}\right)^2}, \quad \psi_\Delta = \operatorname{arc\,tg} \frac{A^* \sin\psi^* - A_1 \sin\psi_1}{A^* \cos\psi^* - A_1 \cos\psi_1},$$

is applied to the controlled selection system, which is intended to separate the fundamental harmonic and to suppress the higher harmonics. From the amplitude of the oscillations of the output of the controlled selection system, $\Delta\bar{u} \sim A_\Delta \sin(\omega t - \psi_\Delta)$, one can judge how well the amplitude and phase of signal u^* coincide with the amplitude and phase of the fundamental harmonic of signal $\underline{u}$: for $A_1 = A^*$ and $\psi_1 = \psi^*$, the amplitude of the selection system's output signal will be minimal. In particular, if the selection system does not pass the higher harmonics at all, or if the investigated system is linear, then the amplitude of the controlled selection system's output signal, for $A_1 = A^*$ and $\psi_1 = \psi^*$, will equal zero. For observing the output signal of the selection system, we use an indicator whose natural frequency is significantly higher than the frequency being investigated. From the output signal of the comparison block, Δu, with total compensation of the fundamental harmonic, one can make a summary estimate of the higher harmonics.

For obtaining the amplitude and phase of higher harmonics, one must provide for the possibility of obtaining, together with the harmonic signal of the fundamental frequency, the signals at multiples of this frequency. If a sine potentiometer, turned by a drive motor, is used as the transducer for the fundamental frequency then, to obtain multiples of this frequency, one can also use a sine potentiometer, one whose shaft is geared to the shaft of the potentiometer for the fundamental frequency through a reducer which would provide a rotational speed which is an integral multiple of the speed of the drive shaft.

If the system being investigated is linear, its frequency characteristics can be determined by means of our instrument, not only for harmonic input signals, but also for such periodic signals as rectangular impulses, sawtooth signals, etc., which follows from the principle of isolating the first harmonic which underlies the operation of the instrument.

Theory of Operation of the Instrument's Basic Blocks

The forming device is a series connection of a phase-shifter and a voltage amplifier (Fig. 3). As is obvious from the forming device's transfer function:

$$\Phi_{u^*}^{u}(p) = K_A\left(\frac{2}{Tp+1} - 1\right) = -K_A \frac{Tp-1}{Tp+1},$$

where $T = RC$ is the time constant of the RC network and K_A is the over-all gain, its amplitude characteristic $A^*/A_0 = K_A$ does not depend on either the frequency ω or the time constant T. The phase lag ψ^*, defined by the formula $\tan(\psi^*/2) = \omega T$, depends only on time constant T and frequency ω. This allows the amplitude A^* and the phase ψ^* of the compensating signal to be controlled separately, which significantly simplifies their tuning.

The value of the amplitude characteristic A^*/A_0 is determined directly by indicator divider D_1, with the gain K_1 of vacuum tube amplifier VTA_2 taken into account. This value equals $K_A = \alpha K_1$, where α is the scale coefficient for divider D_1. (For convenience, the circuit is provided with two possible gains, $K_1 = 1$ and $K_1 = 10$.)

The value of time constant T, necessary for computing the phase shift $\psi^* = 2 \arctan \omega T$, is determined by the scale indication of variable impedance R and switch K_{1C} for capacitors C_1 and $10C_1$. Switching of the capacitors provides a widening of the range of variation of time constant T.

For variations of ω from 0 to ∞, the phase shifter provides phase shifts in the range $0 \le \psi^* < 180°$. There is no difficulty, in principle, in creating still greater phase shifts, either positive or negative, by varying the sign of the signal being investigated.

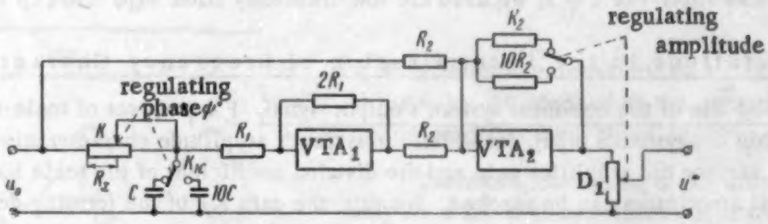

Fig. 3.

The scale block is the conjunction of a vacuum-tube amplifier with a voltage divider, designed to bring the scale of the nonlinear system's output signal to the scale of the input signal u_0. The inclusion of the scale block in the scheme simplifies the operation, since the value of amplitude characteristic K_A, after being thus brought to the proper scale, can be determined directly from the scale of divider D_1 (with account taken of gain K_1 of vacuum-tube amplifier VTA$_2$). The amplifier in the scale block also provides the transformation of the voltage of the output transducer of the system under investigation to the normal working voltage, which is particularly important in case one is measuring a mechanical output biasing by means of tension transducers, or by transducers with low output voltages. In addition, a voltage from a controlled potentiometer is applied to the input of the scale block amplifier for the compensation of the dc component of the investigated signal.

Selection system. As the selection system in the instrument being described here, we use a vacuum tube amplifier with a twin-T bridge in the feedback path (Fig. 4). Such a circuit is a band-pass filter whose passband is defined by the magnitudes of resistors R_1, R_2, and R_3, capacitors C_1, C_2, and C_3, and gain K_0 [8]. For a sufficiently high degree of selectivity (i.e., for a sufficiently narrow passband) this system, tuned to the frequency $n\omega$ of the harmonic being investigated, does not pass, for all practical purposes, any higher or lower harmonics, which permits one to judge, from the absence of oscillations at its output, that the harmonic under investigation is absent from its input signal.

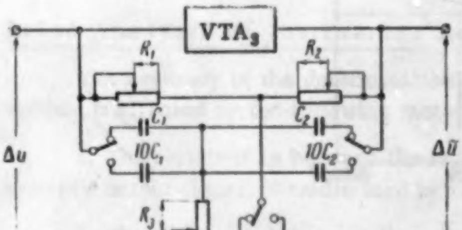

Fig. 4.

Tuning of the selection system to the investigated frequency is carried out by simultaneous variations of resistors R_1, R_2, and R_3 in such manner that the ratios $R_1 : R_2 : R_3 = 1 : 1 : 0.5$ are kept. With these ratios of the resistances, and with capacitor ratios of $C_1 : C_2 : C_3 = 1 : 1 : 2$, the frequency passed by the system, defined by the feedback cut-off frequency, equals $\omega_0 = 1/R_1C_1$. For the use of the selection system, a tenfold change of ω_0 was provided for by the capability of switching condensors C_1, C_2, and C_3. To ease the computation involved in determining the phase shift $\psi^* = 2 \arctan \omega T$, the scale of R_1 was graduated in units of angular frequency. The selectivity of the system described depends on the amplifier gain without feedback, K_0, and will be proportional to K_0. However, an increase in K_0 protracts the transient response when the amplitude and phase of the input signal change. Since the duration of the transient response turns out to be proportional to the quantity $K_0 + 1$, and inversely proportional to the cut-off frequency ω_0 of the feedback, it is not practical at low frequencies to use a large value of K_0. In the selection system being described, it was made possible to increase the selectivity by a transition from low frequencies, $f < 2$ to 5 cps, to higher frequencies, $f > 2$ to 5 cps, by varying gain K_0 in the range $5 \le K_0 \le 25$. The given values of K_0 provide variations in the suppression coefficient of the third harmonic, $A(\omega)/A(3\omega)$, in the range $3.4 \le A(\omega)/A(3\omega) \le 16.4$.

For the determination of the frequency characteristics of a number of nonlinear systems (boosters, steering gear, and other power drives), in which the amplitude of the third harmonic does not exceed 10 to 20% of the amplitude of the fundamental, it is ordinarily sufficient to have the coefficient of suppression of the third harmonic of the order of 5 to 8. In this case, instead of an amplifier with a twin-T bridge in the feedback path, one can also use a second-order oscillatory system with a low damping coefficient, on a low-frequency filter in which the cut-off frequency lies between the fundamental frequency and its triple. Thus, if the oscillatory system has the transfer function $\Phi(p) = \frac{K}{T^2p^2 + 2\zeta Tp + 1}$, it will suffice, for attaining the sensitivity mentioned, that the condition $\zeta = 0.5$ to 1 be met. For $\zeta = 1$, we have the low-frequency filter $\Phi(p) = K/(Tp + 1)^2$.

Sequence of Operations in the Determination of Frequency Characteristics

1. Tuning to the scale of the nonlinear system's output signal. For purposes of scale-tuning, a low-frequency signal is impressed upon the system's input, for which the system's amplitude characteristic may be taken equal to unity and then, by varying the amplifier gain and the division coefficient of the scale block, equality of the input and output signal amplitudes can be reached. For this, the gain K_A of the forming device must be unity.

2. Tuning of the selection system. The selection loop used in the instrument for controlling both frequency and degree of selectivity can also be simultaneously used for measuring the frequency ω. In case the frequency ω of the forced oscillations is not exactly known, it is necessary to tune to the frequency ω by changing variable resistors R_1, R_2, and R_3, while observing the results of this tuning by the amplitude of the selection system's output signal. The frequency measured by the scale of R_1 can be used for an approximate calculation of the phase shift $\psi^* = 2 \arctan \omega T$. For the accurate calculation of the phase shift, it is necessary to determine, with the requisite accuracy, the transducer's fundamental frequency by means of a special instrument, namely, a freqmeter.

3. Tuning the amplitude A^* and the phase ψ^* of the compensating signal. Tuning of compensating signal u^* should first be implemented by phase, and, thereafter, by amplitude, since this significantly shortens the time necessary for the tuning. The necessity for this sequence derives from the following.

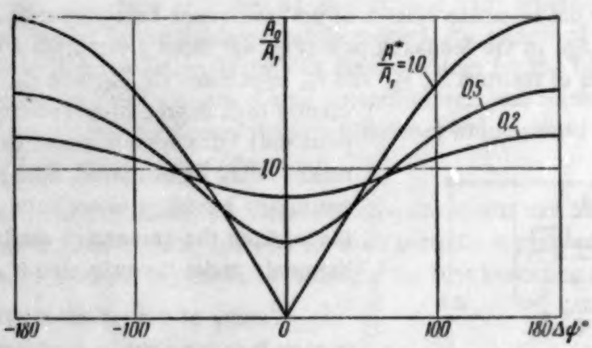

Fig. 5.

The result of tuning the compensating signal is observed by seeing a minimum amplitude of the selection system's output signal $\Delta\tilde{u}$ which, in practice, is proportional to the amplitude A_Δ of the difference

$$u_1 - u^* = A_1 \sin(\omega t - \psi_1) - A^* \sin(\omega t - \psi^*) = A_\Delta \sin(\omega t - \psi_\Delta).$$

To provide the foundation for the tuning sequence given, it suffices to consider the effect of A^* and ψ^* on the amplitude of

$$A_\Delta = A_1 \sqrt{1 - 2\frac{A^*}{A_1}\cos\Delta\psi + \left(\frac{A^*}{A_1}\right)^2} \qquad (\Delta\psi = \psi_1 - \psi^*).$$

As is obvious from the expression for the partial derivative

$$\frac{\partial A_\Delta}{\partial(\Delta\psi)} = \frac{A^* \sin\Delta\psi}{\sqrt{1 - 2\frac{A^*}{A_1}\cos\Delta\psi + \left(\frac{A^*}{A_1}\right)^2}},$$

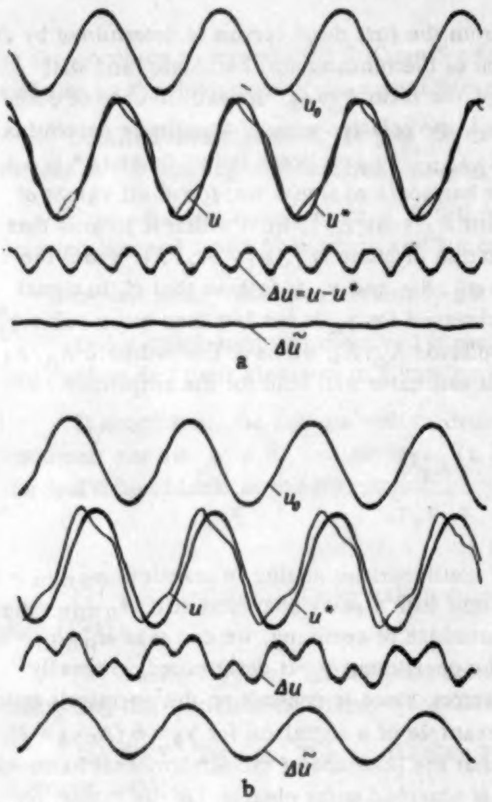

Fig. 6.

the minimum of the amplitude A_Δ as phase ψ^* varies (for a fixed value of A^*) is reached for $\Delta\psi = 0$, i.e. for the coincidence of the phases ψ^* and ψ_1, independently of the magnitude of amplitude A^*. At the same time, it follows from the expression for the partial derivative

$$\frac{\partial A_\Delta}{\partial\left(\frac{A^*}{A_1}\right)} = \frac{\left(\frac{A^*}{A_1} - \cos\Delta\psi\right) A_1}{\sqrt{1 - 2\frac{A^*}{A_1}\cos\Delta\psi + \left(\frac{A^*}{A_1}\right)^2}}$$

that the minimum amplitude of A_Δ as A^* varies is attained at the value A^*, defined by the equation $A^*/A_1 - \cos\Delta\psi = 0$, i.e. depending on the phase difference $\Delta\psi$.

Therefore, the tuning of the amplitude must be done only for a tuned phase. However, if the amplitude of the compensating signal differs significantly from amplitude of the harmonic being investigated, then the curve $A_\Delta = f(\Delta\psi)$ has a very shallow minimum (Fig. 5), which makes an exact tuning of ψ^* difficult. Due to this, the tuning of ψ^* and A^* must be carried out in two steps: first a rough tuning of phase and amplitude and then, in the same order, a refined (Vernier) tuning.

As an example, Fig. 6a and 6b shows the oscillograms of the signals u_0, $\underline{u}$, u^*, Δu and $\Delta\tilde{u}$, obtained in the determination of the frequency characteristics of a nonlinear system with an odd static characteristic, $du/dt = f(u - u_0)$. Figure 6a shows the signals when there was complete coincidence of the compensating signal u^* and the first harmonic signal $\underline{u}$, and Fig. 6b shows them when these signals were mismatched in phase. As is clear from Fig. 6a and 6b, the output signal $\Delta\tilde{u}$ of the controlled selection system does not, in practice, contain higher harmonics so that, from the amplitude of its oscillations, one can easily observe the fact of a mismatch of signal u^* with the first harmonic signal $\underline{u}$.

Brief Analysis of Instrument Accuracy

The accuracy of the determination of amplitude and phase characteristics using the instrument herein described is affected by the following factors:

1. The relationship between the amplitudes of the fundamental and the higher harmonics of the nonlinear system's output signal, characterized by the coefficients $\alpha_n = A_1/A_n$.

2. The values of the suppression coefficients of the selection system

$$\beta_n = \frac{\overline{A}(\omega)}{\overline{A}(n\omega)},$$

where $A(\omega)$ is the amplitude of the selection system at its resonant frequency.

3. The accuracy of the visual observation of the fundamental harmonic in the selection system's output signal $\Delta\tilde{u}$, characterized by the coefficients $\gamma_n = \frac{\tilde{A}_n}{\tilde{A}_\Delta} = \frac{1}{\beta_n}\frac{A_n}{A_\Delta}$, for which the presence of the fundamental harmonic in signal $\Delta\tilde{u}$ is still sufficiently readily observed (here, we denote by $\tilde{A}_0$ and $\tilde{A}_n$ the amplitudes of the fundamental and the greatest n-th harmonic at the selection system's output).

4. The accuracy of determining the original frequency ω.

5. The accuracy of establishing the amplitude A^* and the phase ψ^* of the compensating signal provided by the forming device.

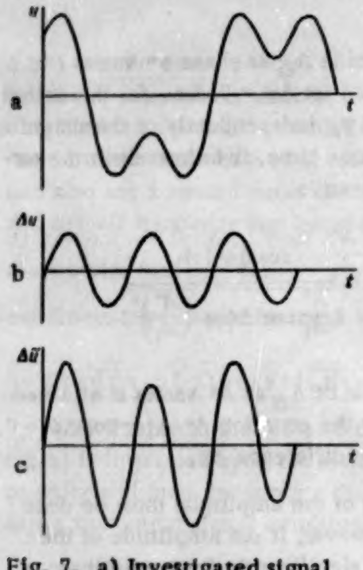

Fig. 7. a) Investigated signal ($A_{n>3} = 0$) and $u = A_1 \sin \omega t + 0.5 A_1 \sin 3\omega t$, $\alpha_3 = 2$; b) input signal to the selection system (difference signal) $\Delta u = 0.007 A_1 \sin \omega t + 0.5 A_1 \sin \omega t$, $A_\Delta/A_1 = 0.007$; c) output signal of the selection system $\Delta\tilde{u} = k(0.2 \sin \omega t + \sin 3\omega t)$, $\beta_3 = 14.4$ and $\gamma_3 = 5$.

The error arising from the first three factors is determined by the accuracy of compensation of the fundamental harmonic, and will decrease proportionally to the ratio A_Δ/A_1. Indeed, in case of coincidence of phase ($\psi^* = \psi_1$), the relative error of amplitude determination is $|A_1 - A^*|/A_1 = A_\Delta/A_1$. For amplitude coincidence ($A^* = A_1$), the amplitude of the first harmonic of signal Δu, for small values of $\Delta\psi$, can be put in the form $A_\Delta = A_1|\Delta\psi|$, from which it follows that the error in the determination of phase is $|\Delta\psi| = A_\Delta/A_1$. From the definition of coefficients α_n, β_n, and γ_n it follows that if, in signal $\Delta\tilde{u}$, the basic harmonic observed for γ_n is not less than some value γ_n^* then, for the relative amplitude A_Δ/A_1, we have the estimate $A_\Delta/A_1 \ll 1/\alpha_n\beta_n\gamma_n$. Analogous estimates will hold for the amplitude and phase:

$$\frac{|A_1 - A^*|}{A_1} \leqslant \frac{1}{\alpha_n \beta_n \gamma_n^*}, \quad |\Delta\psi| \leqslant \frac{1}{\alpha_n \beta_n \gamma_n^*}.$$

For the majority of nonlinearities arising in practice, $\alpha_{n\,\min} > 2$ (for example, for an element with a relay characteristic, $\alpha_{n\,\min} = \alpha_3 \approx 3$). Therefore, for an estimate of accuracy, we can take $\alpha_{n\,\min} = 2$. The minimum value of the coefficient γ_n is determined, generally speaking, by subjective factors, since it depends on the operator's training. Figure 7 shows the example of a signal $\Delta\tilde{u}$ for $\gamma_3 = 5$ ($\bar{A}_{n>3} = 0$), from which it is obvious that the presence of the fundamental harmonic in signal Δu when $\gamma_3 = 5$ is observed quite clearly. In the future, for estimating accuracy, we shall take $\gamma_n^* = 5$.

Suppression coefficient β_n is defined by the amplitude characteristic of the selection system. For the chosen selection system [8]

$$A = K_0 \sqrt{\frac{1 + \left(\frac{4\bar{\omega}}{1 - \bar{\omega}^2}\right)^2}{(K_0 + 1)^2 + \left(\frac{4\bar{\omega}}{1 - \bar{\omega}^2}\right)^2}},$$

where $\bar{\omega}$ is the frequency relative to resonant frequency ω_0.

It follows from this that

$$\beta_n = \frac{A_{\bar{\omega}=1}}{A_{\bar{\omega}=n}} = \sqrt{\frac{(K_0 + 1)^2 + \left(\frac{4n}{1 - n^2}\right)^2}{1 + \left(\frac{4n}{1 - n^2}\right)^2}}.$$

For $K_0 = 25$ and $n = 3$ we have that $\beta_3 = 16.4$. By substituting the values $\alpha_{3\,\min} = 2$, $\gamma_3^* = 5$, and $\beta_3 = 16.4$ in the expressions for the estimates of $|A_1 - A^*|/A_1$ and $|\Delta\psi|$, we find that the maximum error due to the first three factors does not exceed 0.7% for amplitude and 0.4° for phase.

The error in defining the initial frequency $\Delta\omega$ affects only the accuracy of the phase determination. As follows from the expression for the phase shift, $\psi = 2 \arctan \omega T$, the error in phase determination is $\Delta\psi = \frac{d\psi}{d\omega}\Delta\omega = \frac{2T}{1 + (\omega T)^2}\Delta\omega$ or $\Delta\psi = \Delta\omega \sin\psi/\omega$. The maximum error occurs for $\psi = 90°$ and equals $\Delta\psi_{\max} = \Delta\omega/\omega_1$ from whence it follows that the error does not exceed the relative error in frequency measurement $\Delta\omega/\omega$. Existing frequency meters can provide frequency measurements with an error of less than 1%. With this, the error $\Delta\psi$ will not exceed 0.5°.

The accuracy of establishing the phase ψ^* and amplitude A^* provided by the forming device is determined

by the accuracy in reproducing the transfer function $K_A(Tp-1)/(Tp+1)$ by means of the chosen electrical system, and also by the deviations of the actual parameters from the design values.

Detailed investigations, the presentation of which is not possible in this paper, show that the maximum error due to the forming device cannot exceed 0.5° in phase and 1% in amplitude.

From what has been presented, it follows that the total maximum error given by the instrument cannot, in practice, exceed 1 to 1.5° in phase and 1.5 to 2% in amplitude.

Experimental data have backed up the data given here on the accuracy of the instrument's operation.

Today, the instrument described is used for determining frequency characteristics of actual automatic control systems and their elements in a number of undertakings.

In conclusion, the authors wish to express their gratitude to G.V. Aleksandrov for his aid in building the instrument, and also to A.F. Vasilevskaya, Yu.I. Rudakov, and K.V. Pavlov for taking part in the development of the individual blocks of the instrument.

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DIGITAL MODELS

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A systematic survey is made of contemporary literature dealing with a class of new computing devices, namely, digital differential analyzers, digital computers operating by the method of summing increments, and with individual function generators whose input and output quantities are presented by the method of delta-modulation of pulsed electrical voltages. The theoretical bases of the operation of digital models is formulated, and comparisons are made of their characteristics with the characteristics of electronic digital computers and of analog computers. For this, a computing machine is considered as an equivalent filter which transforms an input signal, and as the criterion for comparing different classes of machines, we take the product of the filter's resolving power by the width of the pass-band spectrum.

Recently, there has been heightened interest in a new type of electronic computer, called digital differential analyzers, or machines which operate by summing increments (incremental computers). The fundamental characteristic peculiarity of these machines is the possibility of isolating in their make-up individual functional blocks which execute a number of basic mathematical operations, such as integration, summation, multiplying by a constant factor, etc. For the make-up, or programing, for a problem, the blocks are interconnected in accordance with a scheme which provides a given sequence of operation executions. From the user's point of view, the digital differential analyzers occupy a place intermediate between the universal (general-purpose) digital machines and analog computers, since they join the potential accuracy of digital computers and the simplicity of setting up for a problem characteristic of analog computers.

Independently of digital differential analyzers, a number of devices have recently been developed, and described in the periodical literature [1-24], which employ the methods of presenting variables which have been adopted in digital differential analyzers,and which are designed for the execution of individual mathematical operations, such as multiplication, generating of sine functions, etc. The essential fact here is that such blocks, individually or in definite combinations, can be used for simulation in the true time scale of definite physical processes and, consequently, can operate as elements of automatic control systems. This allows us to unify them under the common denotation of "digital models," and to formulate certain theoretical bases for the operation of such devices.

In digital models, the method of presenting quantities is that known in communication theory as delta-modulation.* In accordance with this method, only the increments of the original variables take part in all the transformations, and only at the final step of these transformations is the resulting quantity obtained in the form of a sum of the individual increments. The scales of all quantities taking part in the transformations are quantized, and the magnitude of an elementary increment is equated to the quantum step δ_{ζ}. Elementary increments

***It is specifically the delta-modulation method which is used, and not a number-pulse code which, in the general case, can be multipositional (cf. for example, A.A. Kharkevich, Outline of General Communication Theory, Gostekhizdat, 1955).**

are presented, in digital models, by individual electrical pulses. The time scale is also quantized and, thus, the presentation of increment pulses can only occur at definite moments of time, t_0, t_1, ... t, separated by the step δ_t of the quantized time scale. The absence of a pulse at a given moment of time t_i has, in one case, the meaning of a negative increment, $-\delta$ (a binary incremental system) and, in other cases, the meaning of zero (the ternary incremental system). For presenting the sign of an increment in the ternary system, an additional pulse, transmitted along a second channel, is used.

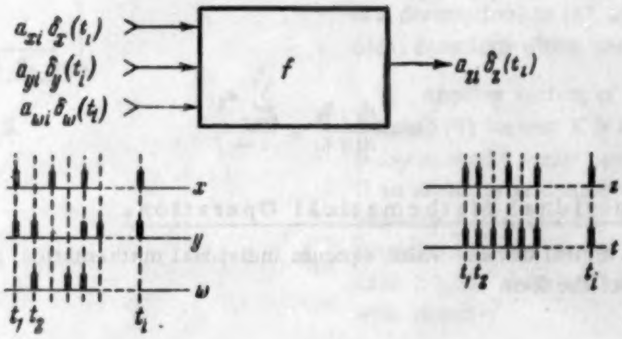

Fig. 1.

Any information transformer which operates with quantities presented by the delta-modulation method receives, at its inputs, successions of increments of the form

$$
\begin{aligned}
&a_{x0}\,\delta_x(t_0),\ a_{x1}\,\delta_x(t_1),\ \ldots,\ a_{xi}\,\delta_x(t_i),\ \ldots,\\
&a_{y0}\,\delta_y(t_0),\ a_{y1}\,\delta_y(t_1),\ \ldots,\ a_{yi}\,\delta_y(t_i),\ \ldots,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
&a_{w0}\,\delta_w(t_0),\ a_{w1}\,\delta_w(t_1),\ \ldots,\ a_{wi}\,\delta_w(t_i),\ \ldots,
\end{aligned}
\tag{1}
$$

where the subscript on a δ indicates the quantity being presented, and the coefficients $a_{\delta i}$ can take the values of +1 and −1 in the binary system, and +1, 0, and −1 in the ternary system. At the information transformer's output there is also formed a series of increments, of the form (Fig. 1)

$$a_{z0}\,\delta_z(t_0),\ a_{z1}\,\delta_z(t_1),\ \ldots,\ a_{zi}\,\delta_z(t_i),\ \ldots \tag{2}$$

The operation of an information transformer can be described, generally speaking, by the corresponding relationship between the original quantities, x, y, ..., w, and the resulting quantity z:

$$z = f(x,\ y,\ \ldots,\ w). \tag{3}$$

Here, however, there is a definite inconvenience, related to the fact that the "instantaneous" values x_i, y_i, ..., w_i and z_i are obtained as the result of the accumulation of increments during the entire time of operation of the transformer and, consequently, the relationships between these values are integral characteristics, and do not describe the operation of the transformer at each given moment of time. For describing transformer operation, it is more convenient to use some linear function of the x, y, ..., w which does not possess the disadvantage just noted. Ordinarily, as such functions, we choose the derivatives with respect to time of the corresponding quantities, characterizing the speed of their change and, to describe the operation of information transformers, we use relationships of the form

$$\frac{dz}{dt} = f\left(\frac{dx}{dt},\ \frac{dy}{dt},\ \ldots\ \frac{dw}{dt}\right). \tag{4}$$

For quantities presented by the delta-modulation method, where quantized amplitude and time scales are used, approximate values of derivatives with respect to time are defined as

$$\frac{d\xi}{dt} \approx \frac{\Delta\xi}{\Delta t} \approx \frac{\sum_{k=j}^{i} a_{\xi k}\, \delta_\xi (t_k)}{t_i - t_j}, \quad i > j. \tag{5}$$

The step δ_t of the quantized time scale is, as a rule, the same for all the transformers involved in the solution of a given problem. The quantization step δ_ξ is chosen for an established scale of the variables. With these conditions, the approximate normalized value of the speed of change of variable ξ can be defined by the expression

$$v = \frac{d_\xi / \delta_\xi}{dt / \delta_t} \approx \frac{\sum_{k=j}^{i} a_{\xi k}}{i - j}. \tag{6}$$

Blocks Executing Individual Mathematical Operations

The simplest form of digital devices which execute individual mathematical operations are the devices which realize relationships of the form

$$\frac{dz}{dt} = K\frac{dx}{dt}, \tag{7}$$

where K is a constant. For the case when the increments are only positive (the variant of the binary system of increments in which the coefficients can take only the values of 0 and 1), but K = 0.5, the role of such a transformer can be played by an ordinary trigger (flip-flop). Analogously, a series connection of two, three, etc. triggers can realize relationship (7) for the cases when K = 0.25, 0.125, etc. Using triggers, we can construct circuits which realize relationship (7) for any K of the form

$$K = 1/n, \tag{8}$$

where n is an integer. A circuit for n = 1, 2, ..., 16 is shown in Fig. 2. Here, we have used the convention that the 0 state in each of the four triggers (denoted by Tr on Fig. 2) corresponds to the conducting state of the right vacuum tube, and state 1 corresponds to the conducting state of the left vacuum tube. Excitation of each following cell is produced by negative pulses (i.e. at the moment of triggering a tube, whose anode is joined to the feedback path). The output pulse, via the feedback path, resets all cells to zero. The fact that any of the keys (contacts) K_1-K_4 is in its lower position is described by the symbol 1, the symbol 0 being used when a key is in its upper position.

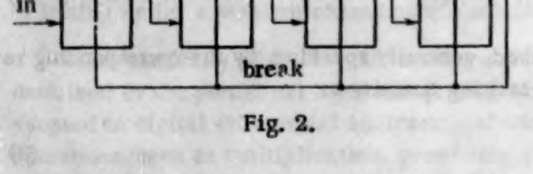

Fig. 2.

The combination 1111 of key positions corresponds to the standard circuit of inter-stage connections for a binary counter. If we assume that a continuous sequence of electric pulses is applied to the circuit's input, then we shall have the following sequence of combinations of trigger states: 0000, 1000, 0100, 1100, ..., 1111. The negative output pulse will appear each time the circuit makes the transition from state combination 1111 to state combination 0000, i.e., once for each 16 input pulses (n = 16). The combination 0000 of key positions would, with no feedback path, correspond to the sequence of combinations of trigger states: 0000, 1111, 0111, etc. However, thanks to the feedback path, each input pulse gives rise to the appearance of an output pulse, and returns the circuit to state 0000 (n = 1). With the feedback path switched out, the combination 1100 of key positions would correspond to the sequence of trigger states: 0000, 1000, 0100, 1100, 0011, etc. With the feedback path closed, an output pulse returning the circuit to its initial state would appear at the moment when combination 0011 appeared. This corresponds to the case n = 4. From a further consideration of the circuit of Fig. 2, we can immediately draw the conclusion that such a circuit, with an arbitrary number of triggers, will realize relationship (7) with the condition that

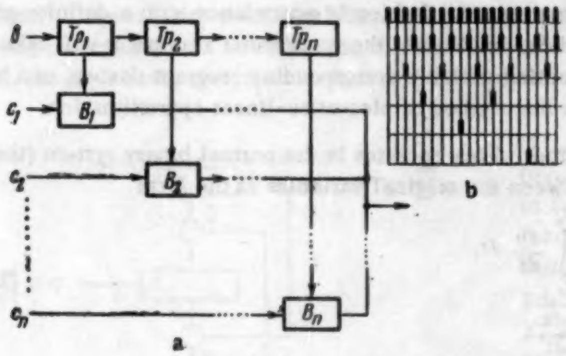

Fig. 3.

$$n = 1 + \sum_{i=1}^{m} b_i \cdot 2^{i-1}, \tag{9}$$

where the b_i are coefficients assuming the values of 1 or 0 and which characterize, in the notation adopted earlier, the states of the corresponding keys. Circuits of this type are described in more detail in [6]. There is a description, in [4], of a circuit for generating sinusoidal functions which uses an analogous method.

Another variant of the circuit for realizing relationship (7) for any K is shown in Fig. 3. Here, there is again used a series connection of triggers, $Tr_1 - Tr_n$. If an arbitrary sequence of pulses, characterized by velocity $\underline{v}$ [the velocity is defined by formula (6)], is impressed on the circuit's input then, at the output of each trigger, there will appear a sequence of pulses with velocity

$$v_i = \frac{v}{2^i} \qquad (i = 1, 2, \ldots, n), \tag{10}$$

where $\underline{i}$ is the ordinal number of the trigger. For this, the output pulses are connected to the anodes of the corresponding triggers in such fashion that no pair of pulses from any pair of triggers coincides in time (Fig. 3b). If this latter condition is met, then the velocity of the pulses at the circuit's output will be defined by the relationship

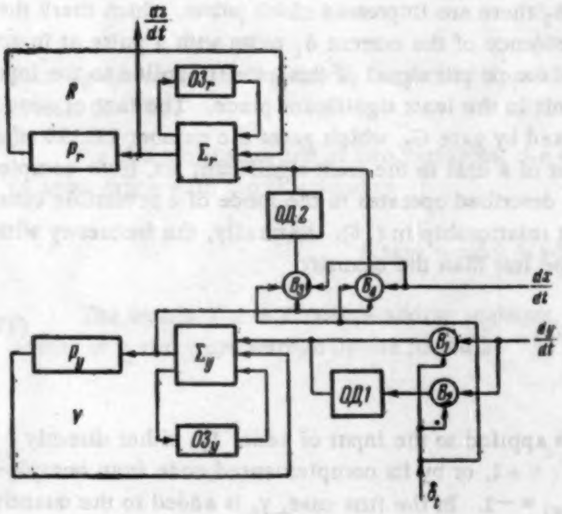

Fig. 4.

$$v_{\text{out}} = \sum_{i=1}^{N} c_i v \cdot 2^{-i} \tag{11}$$

where the coefficients c_i characterize the states of the corresponding gates G_i (a value of 1 for a coefficient $\underline{c}$ corresponds to an open gate G_i, while the value of 0 corresponds to a closed gate), and $\underline{n}$ is the total number of triggers. Here, consequently,

$$K = \sum_{i=1}^{n} c_i \cdot 2^{-i} \tag{12}$$

In both circuits considered, the quantity K can, generally speaking, be some function of the quantity $\underline{x}$:

$$K = K(x). \tag{13}$$

In this case, relationship (7) can be rewritten in the form

$$dz = K(x)dx \tag{14}$$

$$z = z_0 + \int_{x_0}^{x_0} K(x)\,dx. \tag{15}$$

Thus, the circuits described are blocks for multiplication by a constant, when K is constant (for K = const, it follows immediately from (7) that $z = Kx + z_0$), and integrators when K = K(x). The case is also possible when

K is a function of the independent variable x, but varies in a stepwise fashion in accordance with a definite program as x reaches the given values $x_1, x_2, \ldots, x_i$. Under these conditions, the transformer's output z will take the form of a broken line, and the transformer itself, in conjunction with the corresponding program device, can be used for generating arbitrary functions of the variable x by the method of piecewise-linear approximation.

Figure 4 shows the circuit of an information transformer which operates in the normal binary system (the system of +1 and −1) and which realizes relationships between the original variables of the form

$$y_i = y_0 + \int_{t_0}^{t_i} \frac{dy}{dt}\, dt, \qquad \frac{dz}{dt} = Ay_i \frac{dx}{dt}, \tag{16}$$

Here, systems Y and R are delay lines or shift registers (SR_y and SR_r) whose constants are numbers, coded in binary (or any other number system), which are continuously circulating through the circuits closed through the one-place adders Σ_y and Σ_r. At the circuit's input denoted by δ_t there are impressed clock pulses, which mark the successive moments of time $t_0, t_1, \ldots, t_i$. The fact of coincidence of the current δ_t pulse with a pulse at input dy/dt (a_{yi} + 1 in the binary system) is fixed by gate G_1, and the output signal of this gate is applied to the input of adder Σ_y, incrementing the contents of register Y by a unit in the least significant place. The fact of noncoincidence of a δ_t pulse with a pulse at the dy/dt input is fixed by gate G_2, which gates the number 11...10 (the least significant order is on the right), the one's complement of a unit in the least significant bit, from complement former CF_1 to the input of adder Σ_y. The system just described operates in the mode of a reversible counter, accumulating the current value of y_i, and realizing the first relationship in (16). Naturally, the frequency with which the orders follow each other in the register must not be less than the quantity

$$f = N/\delta_t, \tag{17}$$

where N is the capacity of the register.

The current value of y_i from the output of adder Σ_y is applied to the input of adder Σ_r, either directly through gate G_4, if the current increment of variable x is $a_{xi} = +1$, or by its complemented code from complement former CF2 if the current increment of variable x is $a_{xi} = -1$. In the first case, y_i is added to the quantity contained in register R and, in the second case, is subtracted from it. If the current value r_i exceeds the capacity of register R, which is equal to the capacity of register Y, then at the output, dz/dt, of the circuit there appears a pulse which is simply the overflow pulse of register R. If the capacities of registers Y and R have the common value N, then the quantity y_i can be given in the form

$$y_i = 2^N/m, \tag{18}$$

where $m \geq 1$, since $y_i \leq 2^N$. It is clear from (18) that, to obtain one overflow signal from register R, it is necessary to transmit the contents of register Y m times to register R. By taking into account that each such transmission is executed under the stimulus of the current increment of the quantity x, we obtain the relationship

$$\frac{dz}{dt} = \frac{1}{m}\frac{dx}{dt} = 2^{-N} y_i \frac{dx}{dt},$$

which is the second relationship of (16) if we take $A = 2^{-N}$. Relationship (18), the validity of which for positive increments and positive values of y_i and r_i is obvious, will also be valid for any r_i, y_i, and a_{xi} if negative values of r_i and y_i are presented in the complemented code, and if the most significant bits of r_i and y_i (this position is made binary even when the numbers r_i and y_i are coded in some other number system) play the role of sign bits, where a 1 in this bit corresponds to plus. This statement can be verified by a consideration of the corresponding examples (cf., [1]). In an analogous way, one can construct a circuit operating in the ternary incremental system.

The circuit of Fig. 4, just as the previous ones, can be used for multiplying the speed of variation of quantity x by a constant factor, for integrating and for generating functions by the method of piecewise-linear approximation. Moreover, the circuit can be used for one more special operation. Since, in the given case, the overflow signal from register Y is not used, but the presence of such an overflow might, naturally, lead to an erroneous

result, measures are usually taken to stop the computer if the Y register overflows. However, if such a step does not, for some reason or other, occur, then an overflow of the register by one unit in the least significant bit leads to the replacement of the maximum possible value of y, which was stored in this register prior to the reception of the unit in the least significant bit which engendered the overflow (11...1 in the binary system), by its minimum possible value (00...0 in the binary system). Conversely, subtraction of a unit in the least significant bit from the minimum possible value of y leads to the formation of its maximum possible value. Due to this property, the circuit of Fig. 4, with the condition that the minimum or maximum value of y is initially placed in register Y, can operate as an independent relay amplifier, in which a variation in the contents of register Y by one unit in the least significant place leads to a change in the velocity of the output signal from plus the maximum to minus the maximum. Ordinarily, in this case, a signal with the maximum velocity (signal δ_t) is applied to input dx/dt of the circuit. Such an amplifier is used for forming discontinuous functions, for setting up adder circuits and, in a number of other special cases (for more details cf. [1]).

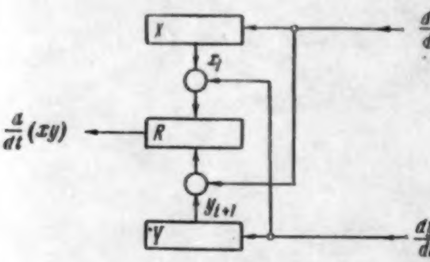

Fig. 5.

For the multiplication of two variables, the circuit shown on Fig. 5 is used. Multiplication is implemented in accordance with the relationship

$$\Delta (xy) = x\Delta y + y\Delta x + \Delta y\Delta x = x\Delta y + (y + \Delta y)\Delta x. \tag{19}$$

The letters X and Y denote adding registers, analogous to adding register Y on Fig. 4, in which the current values of x and y are formed by the formulas

$$x_i = x_0 + \int_{t_0}^{t_i} \frac{dx}{dt}\, dt,$$
$$y_i = y_0 + \int_{t_0}^{t_i} \frac{dy}{dt}\, dt. \tag{20}$$

Letter R denotes a summing register which is also analogous to register R of Fig. 4. In each cycle of circuit operation, quantity x is applied to register R (by the direct or complemented code, depending on the value of a_{y_i}) until the current increment (denoted by x_i on the circuit) has been added to it, and the quantity y is applied to the Y register after the current increment has been added ($y_{i+1} = y_i + \Delta y$). The overflow signals from register R are the increments of the product, in correspondence with (19).

The Construction of Digital Differential Analyzers

For the solving of problems, the individual functional blocks of a digital model are interconnected in correspondence with the initial equations in exactly the same way as in electronic analog computers. There are two basic types of digital differential analyzers today: parallel and serial. A parallel digital differential analyzer is a set of an arbitrary number of functional blocks, each of which constitutes an independent physical package. Such a machine contains a common power supply and a common source of basic synchronizing pulses. For a problem setup, the blocks are interconnected by cables.

In serial digital differential analyzers, the contents of registers Y and R of each integrator are written successively in common delay lines with capacity

$$Q = MN, \tag{21}$$

where N is the capacity of one integrator register and M is the number of integrators in the machine. The circuit of a series digital differential analyzer is fundamentally analogous to the circuit of Fig. 4, with the difference

TABLE

Name and year of production	MADDIDA 1950	CRC — 101 1950	CRC — 105 1952	DA — 1 1954	D — 12 1954	D — 18 1956	G — 15 1954
Company	Northrop Aviation, USA	Computer Research Corporation, USA	Computer Research Corporation, USA	Bendix, USA	Bendix, USA	Bendix, USA	Bendix, USA
Type	Serial	Serial	Serial	Serial	Serial	Parallel	General purpose electronic digital computer
Number system, word length	Binary	Binary 22	Decimal 6	Binary 29	Decimal 7	Decimal 7	Binary 29
Increment system and integration method	Binary $dz = ydx$	Binary $dz = ydx$	Ternary $dz = ydx$	Ternary —	Ternary $dz = \left(y + \frac{1}{2}dy\right)dx$	Binary Extrapolation	—
Number of functional blocks	22	22	60	108	60	—	—
Speed, in iterations/sec	—	60	60	30	100	20 000	Addition 0.54 msec
Size, weight	0.162 m^3	—	—	—	3 m^3 908 kg	—	0.81 m^3
Power requirements	—	—	—	—	6 kva	—	3 kva
Number of tubes, transistors, diodes	—	—	—	—	—	—	—

TABLE (continued)

Name and year of production	Litton 20	ADA 1957	TEDDA 1957	TRICE 1958	ADDAMII 1958	Airborne Rocket plane X-10	C — 1100 Airborne
Company	Litton, USA	Australian Institute of Engineers	Tokyo, Shibaura Electric, Japan	Paccard Bell, USA	A. V. Roe. England	North American Aviation, USA	Philco Corporation, USA
Type	Serial	Serial	Serial	Parallel	Parallel	Serial	General purpose electronic digital computer
Number system, word length	Binary	Decimal 6	—	Binary 30	Binary 14	—	—
Increment system and integration method	—	Ternary $dz = \left(y + \frac{1}{2}dy\right)dx$	Ternary $dz = ydx$	Ternary $dz = \left(y + \frac{1}{2}dy\right)dx$	Ternary $dz = ydx$	—	—
Number of functional blocks	20	50	60	—	80	93	—
Speed, in iterations/sec	—	50	—	10^5	Approximately 10 000	—	—
Size, weight	—	—	—	One block 6 dm^3	0.42 m^3	0.081 m^3 66 kg	—
Power requirements	—	100 w	—	—	50 w	100 w	—
Number of tubes, transistors, diodes	—	1300 transistors 5000 diodes	—	120 transistors 430 diodes in one block	—	—	3900 transistors 2300 resistors

that the capacity of the registers is defined by (21). Here, at each moment of time, the operations in some one integrator are carried out, so that, for the execution of the operations in all the integrators, a time interval of

$$\delta_t = M\tau, \tag{22}$$

is required, where τ is the time of execution of one elementary operation in one integrator. The overflow signals obtained at the end of each elementary operation in each integrator are written in a special register, and are directed from there to the inputs of other integrators in accordance with a program, introduced into the machine before initiation of problem solution and also stored in special registers. Ordinarily, the tracks on a magnetic drum are used as the registers of series digital differential analyzers.

The basic data on the existing types of digital differential analyzers are given in the table. For comparison purposes, we have included in this table the data on two general-purpose digital computers of the corresponding classes.

Accuracy of Digital Models

The question as to the accuracy of digital models is still insufficiently studied. The basic error sources in solving a problem on a digital model are, naturally, the quantization errors. Some decrease in these errors is attained by operating in the ternary incremental system, and also by the replacement of approximate integration by the rectangular formula (16) by integration by the trapezoidal formula

$$dz = A\left(y_i + \frac{1}{2}\Delta y\right)dx. \tag{23}$$

Moreover, the accumulation of quantization errors in the execution of operations on two variables in a digital model depends on the phase relationships between successive pulses representing each of these variables. This phenomenon is called phase errors. The process of phase error formation can be elucidated with the following example. In the register of some integrator, let there be stored zero which, in the corresponding number system, will have the form 100... At inputs dx/dt and dy/dt of this integrator, let there be applied signals of the form 0, 1, 0, 1, ..., 0, 1, with zero velocity. The contents of register Y will change in the following sequence: 100, ..., 011, ..., 100, ..., etc. With this, the sequence of numbers 011..., 011..., 011... etc. will be applied to register R. The resulting error will obviously equal

$$\varepsilon = \int_{t_0}^{t_i} \delta_y \, dt. \tag{24}$$

This same error can be made equal to zero if the sequence of dy/dt signals has, as before, the form 0,1, 0,1, ..., while the sequence of dx/dt signals is replaced by 1,0, 1,0, ... The phase error is particularly marked in cases when the functional blocks are connected in a circuit shunted by a feedback path.

Rapidity of Digital Models

An estimate of the rapidity of digital models can be made on the basis of the following considerations. Let it be necessary, as the result of solving a given problem, to obtain some function of time x(t) (periodic or admitting an artificial periodic extension) which, with sufficient accuracy, can be presented in the frequency interval $0 \leq \omega \leq \omega_{max}$ by the Fourier series

$$x(t) = x_0 + \sum_{i=1}^{n} A_i \sin(\omega_i t + \varphi_i) = \sum_{i=0}^{n} x_i, \tag{25}$$

each of whose harmonics will be presented in the machine by its derivative with respect to time

$$\frac{dx_i}{dt} = A_i \omega_i \cos(\omega_i t + \varphi_i). \tag{26}$$

The quantity x_i can be obtained in the digital model with some relative error ϵ only in case the selected system of scales satisfies the relationship

$$\frac{A_i}{\delta_{x_i}} \geqslant \frac{1}{\epsilon}. \tag{27}$$

In accordance with (26), the product $A_i\omega_i/\delta x_i$ determines the greatest velocity of increment application necessary for forming the current value x_i. In digital models it must obey the obvious condition

$$\frac{A_i}{\delta_{x_i}}\omega_i \leqslant \frac{1}{\delta_t}. \tag{28}$$

From (27) and (28) we obtain

$$\frac{\omega_i}{\epsilon} \leqslant \frac{1}{\delta_t}. \tag{29}$$

In the literature, the quantity $1/\delta_t$ is usually measured by the number of "iterations" per second where, by iteration, one understands the time interval δ_t during which one elementary operation is executed in all blocks of the machine. For given ϵ, the quantity

$$f_{\max} = \frac{\omega_{\max}}{2\pi} \leqslant \frac{\epsilon}{2\pi\delta_t} \tag{30}$$

characterizes the passband of a low-frequency filter equivalent to the given machine. In serial digital differential analyzers, the quantity $1/\delta_t$ is of the order of 50 to 100 and, in parallel machines, goes as high today as 10^5. With an accuracy of 0.01% for $f_{\max}$ one obtains, respectively, $1.5 \cdot 10^{-3}$ cps for series machines, and 1.5 cps for parallel machines.

Today, electronic computers are conventionally divided into two basic classes — digital and analog. If we accept the definition suggested by Yu.Ya. Basilevskii* for a mathematical machine as a "device which operates in accordance with a system of noncontradictory logical rules, accepts initial information, transforms it in accordance with a given algorithm and then puts out the resulting information," then a natural criterion for placing a given machine in one of the two classes will be the method used in it for coding information. The numerical codes used in digital machines are uniform codes with relatively low radices (almost always 2, 8, or 10) and high positionality. In analog machines (continuous or discrete), codes are used with very high radices and low positionality (usually 1).

The deltamodulation method falls in the second group of codes and, on this basis, digital models must be assigned to the class of analog computers. The relatively high (as compared with the majority of other machines of this class) accuracy of digital models is explained by the high noise stability characteristic of the deltamodulation method, and the simplicity of exact reciprocal transformations of the deltamodulation code and numerical codes, which guarantee accuracy in introducing initial data and in outputing results.

The correctness of assigning digital models to the class of analog devices (and not isolating them, for example, in an independent class) is borne out by a number of additional considerations. The method of preparing and setting up for a problem solution on a digital model is almost completely analogous with the corresponding methods used in operations with mechanical differential analyzers, and very close to the methods applied in operations with electronic models (analog computers). One of the most characteristic features of general-purpose digital computers is the possibility of executing individual operations, or small cycles of operations, whose results can depend as little as desired on the previous and following results. This capability is used for solving such logical problems as, for example, the translation of text from one language to another. In contradistinction to this, the solution of a problem on a digital model is obtained only as the sum of the results of a large number of elementary operations. Finally, the presentation of the program in the form of codes sorted in the device's operational memory, and transformed together with the rest of the information, gives the general-purpose digital machine the ability to modify its programs which is completely lacking in digital models.

* Questions in the Theory of Mathematical Machines. Vol. 1 (edited by Yu. Ya. Basilevskii) [in Russian] (Fizmatgiz, 1958).

The logical operations cited in the literature [1, 2] which are executed by digital models do not go beyond conditional transfers from one set of apparatus to another, or variations of initial conditions, which are also completely realizable in ordinary analog machines.

If one takes into account the heterogeneity of the functional blocks of digital models, and the comparatively broad class of problems solvable by them [1], one will apparently be correct in classing them as general-purpose analog machines. A strict distinction should also be drawn between digital models and special-purpose digital computers. Thus, for example, the "digital differential analyzer" which realizes the iterative process of solving differential equations, as described in [1], is a special-purpose digital machine, since all the information in it is coded in a numerical code.

SUMMARY

1. Digital models are special forms of analog computers in which the quantities which present the original quantities of the problem are deltamodulation voltage pulses.

2. In comparison with electronic analog computers, digital models have a much higher potential accuracy with a lower speed of action. The quantity ω_{max}/ϵ has the same order of magnitude for digital models as for analog computers.

3. As concerns amount of apparatus and power requirements, digital models are roughly analogous to electronic digital computers of the corresponding classes, although in parallel digital models, with their building-block design, the amount of apparatus depends on the complexity of the problem to be solved. Although differing advantageously from general-purpose computers in the greater simplicity of programing problems for them, digital models have significantly less flexibility than general-purpose computers.

4. Slowly flowing processes can advantageously be simulated on digital models.

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PROCEEDINGS OF THE MOSCOW SEMINAR ON THE MAGNETIC ELEMENTS OF AUTOMATION DURING THE YEAR 1959

G.V. Subbotina

During the first half of 1959, the Moscow Seminar on magnetic amplifiers and contactless magnetic elements of automation, remote control and computing technology for the Institute of Automation and Remote Control of the AN SSSR continued its work.

The session of January 28th consisted of the paper of M.A. Boyarchenkov, "Reversible dc instruments with magnetic amplifiers." In considering the possible variants of control schemes for dc motors, the speaker based his work on the developed circuit with combined control of the magnitude of the armature current, by means of saturated reactors, and of the magnitude and direction of the exciter current. For this purpose, a two-cycle fast-acting magnetic amplifier was provided in the exciter circuit.

On the 11th of February, N.P. Vasil'eva gave a paper on "Magnetic logical elements." The paper presented the state of the work being conducted abroad on magnetic logical elements, gave a classification of the known systems of logical elements and presented the results of the investigations and developments at the IAT AN SSSR in systems of logical elements of the types "and," "or," "not," and "memory."

On February 25th, R.A. Lipman and I.B. Negnevitskii gave the communication, "Questions in the design of single-cycle magnetic amplifiers with internal feedback and dc loads." The speakers suggested carrying out the computations for the gain of this type of amplifier by using experimental demagnetization curves.

On March 11th, M.S. Libkind spoke on the development of a new type of three-phase ferromagnetic device for the control of ac power circuits without distortion of the form of the current curves.

On March 25th, A.L. Pisarev, in the paper "Investigation of three-phase magnetic amplifier circuits," spoke of the results of the analysis of the possible types of six-diode, three-phase magnetic amplifier circuits, and considered their special features. The paper contained recommendations as to the use of specific circuits.

The session of April 8th consisted of the presentation of the paper of K.D. Mart'yanova, "Temperature characteristics of industrial magnetically soft alloys."

Two papers were given on April 22nd. In the paper, "The use of magnetic amplifiers for transforming single-phase current to three-phase current and vice versa," A.M. Bamdas and V.A. Kulinich spoke of the development and analysis of the operation of original static electromagnetic transformers. In the paper of A.M. Bamdas and S.V. Shapiro, "New power actuating stabilizers with transformers controlled by shunt magnetization," results in the development of actuator stabilizers were discussed.

On May 13th, V.P. Molomin spoke of the work "Controlling three-phase asynchronous motors by means of magnetic amplifiers from a single-phase network."

On May 27th, V.S. Volodin and M.A. Rozenblat spoke on the development of sources of stable dc voltages based on magnetic and semiconducting elements.

LIST OF LITERATURE ON QUESTIONS OF MATHEMATICAL SIMULATION (BY ANALOG COMPUTERS) FOR 1957 (continued)

Compiled by E.O. Vil'td and R.S. Landsberg

Edited by Cand. Tech. Sci. B.Ya. Kogan

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SIGNIFICANCE OF ABBREVIATIONS MOST FREQUENTLY ENCOUNTERED IN SOVIET TECHNICAL PERIODICALS

AN SSSR	*Academy of Sciences, USSR*
FIAN	*Physics Institute, Academy of Sciences USSR*
GITI	*State Scientific and Technical Press*
GITTL	*State Press for Technical and Theoretical Literature*
GOI	*State Optical Institute*
GONTI	*State United Scientific and Technical Press*
Gosénergoizdat	*State Power Press*
Gosfizkhimizdat	*State Physical Chemistry Press*
Goskhimizdat	*State Chemistry Press*
GOST	*All-Union State Standard*
Gostekhizdat	*State Technical Press*
GTTI	*State Technical and Theoretical Press*
IAT	*Institute of Automation and Remote Control*
IF KhI	*Institute of Physical Chemistry Research*
IFP	*Institute of Physical Problems*
IL	*Foreign Literature Press*
IPF	*Institute of Applied Physics*
IPM	*Institute of Applied Mathematics*
IREA	*Institute of Chemical Reagents*
ISN (Izd. Sov. Nauk)	*Soviet Science Press*
IYap	*Institute of Nuclear Studies*
Izd	*Press (publishing house)*
LÉTI	*Leningrad Electrotechnical Institute*
LFTI	*Leningrad Institute of Physics and Technology*
LIM	*Leningrad Institute of Metals*
LITMiO	*Leningrad Institute of Precision Instruments and Optics*
Mashgiz	*State Scientific-Technical Press for Machine Construction Literature*
MGU	*Moscow State University*
Metallurgizdat	*Metallurgy Press*
MOPI	*Moscow Regional Pedagogical Institute*
NIAFIZ	*Scientific Research Association for Physics*
NIFI	*Scientific Research Institute of Physics*
NIIMM	*Scientific Research Institute of Mathematics and Mechanics*
NIKFI	*Scientific Institute of Motion Picture Photography*
NKTM	*People's Commissariat of the Heavy Machinery Industry*
Obrongiz	*State Press of the Defense Industry*
OIYaI	*Joint Institute of Nuclear Studies*
ONTI	*United Scientific and Technical Press*
OTI	*Division of Technical Information*
OTN	*Division of Technical Science*
RIAN	*Radium Institute, Academy of Sciences of the USSR*
SPB	*All-Union Special Planning Office*
Stroiizdat	*Construction Press*
URALFTI	*Ural Institute of Physics and Technology*
TsNIITMASh	*Central Scientific Research Institute of Technology and Machinery*
VNIIM	*All-Union Scientific Research Institute of Metrology*

NOTE: Abbreviations not on this list and not explained in the translation have been transliterated, no further information about their significance being available to us — *Publisher.*

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